STAT 234 Lecture 1
Probability

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Outline

Coverage: Section 2.1-2.2 of MMSA

- Probability
- Sample Space and Events
- General Addition Rule
- The Complement Rule

What is Probability?

- People talk about *probabilities* or *chances* in our daily life:
  - “What’s the probability that it’ll rain tonight?”
  - “What is the chance that Chicago Cubs make the playoffs this year?”
- For scientific purposes, we need to be more specific in terms of defining and using probabilities

Frequentist Interpretation of Probability

- The probability of heads when flipping a fair coin is 50%
- The probability of rolling a 1 on a 6-sided fair die is 1/6

Everyone agrees with these statements, but what do they really mean?

The *frequentist interpretation of the probability* of an event occurring is defined as the long-run proportion of time that it would happen if we were to repeat the random process over and over again under the same conditions

- Therefore, probabilities are always between 0 and 1

Section 2.1 Sample Space & Events

Sample Space & Events

- The *sample space* (S) of a random phenomenon is the set of all possible outcomes of the random phenomenon.
- An *event* is a subset of the sample space.

Example: If one were to flip a coin 3 times and record the side facing up for each flip,

- the sample space is
  S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
- Events:
  - {all heads} ={HHH}
  - {get one heads} ={HTT,THT,TTH}
  - {get at least two heads} ={HHT,HTH,THH,HHH}

Unions, Intersections, and Complements

- Some events are derived from other events:
  - Rolling a 2 or 3
  - Patient who receives a therapy is relieved of symptoms and suffers from no side effects
- The event that **either A or B occurs** is called the *union* and is denoted $A \cup B$ or *(A or B)*
- The event that **both A and B occur** is called the *intersection* and is denoted $A \cap B$ or *(A and B)*

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- The event that **both A and B occur** is called the *intersection* and is denoted $A \cap B$ or *(A and B)*
  - e.g., {symptoms relieved and no side effects} is {symptoms relieved} ∩ {no side effects}

The Complement of an Event

The event that A **does not occur** is called the *complement* of A and is denoted A^c or *(not A)*

“Empty” Event

Disjoint Events (= Mutually Exclusive Events)

Disjoint (mutually exclusive) events cannot be both true.

- Tossing a coin once, the events {getting H} and {getting T} are disjoint
- The events {John passed STAT 234} and {John failed STAT 234} are disjoint
- Drawing a card from a deck, the events {getting an ace} and {getting a queen} are disjoint

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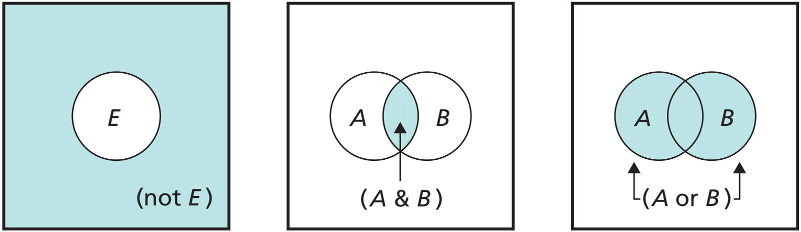
Non-disjoint events can be both true.

- The events {John got an A in STATs} and {John got an A in Econ} are NOT disjoint

Venn Diagrams

Complements, intersections, unions of events can be represented visually using *Venn diagrams*:

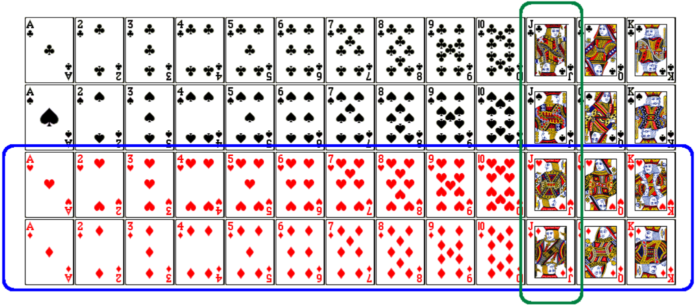
Complement	Intersection	Union
E^c	$A \cap B$	$A \cup B$



Section 2.2 Probability Rules

General Addition Rule

What is the probability of drawing a jack or a red card from a well shuffled full deck?



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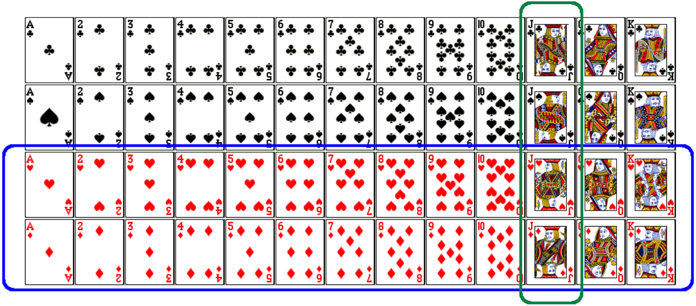

$$P(jack\ or\ red) = P(jack) + P(red) - P(jack\ and\ red)$$

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$$

General Addition Rule

$$P(A \text{ } or \text{ } B) = P(A) + P(B) - P(A \text{ } and \text{ } B)$$

For disjoint events $P(A \text{ } and \text{ } B) = 0$, so the above formula simplifies to

$$P(A \text{ } or \text{ } B) = P(A) + P(B).$$

The Complement Rule

The Complement Rule

- Because an event must either occur or not occur,

$$P(A) + P(A^c) = 1$$

- Thus, if we know the probability of an event, we can always determine the probability of its complement:

$$P(A^c) = 1 - P(A)$$

- This simple but useful rule is called the *complement rule*

Example — The Complement Rule

Question: What is the probability of getting at least one head in 3 tosses of a fair coin?

- Sample space
  $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

  $$\begin{aligned}\text{Event } A &= \{\text{at least one heads}\} \\ &= \{HHH, HHT, HTH, THH, HTT, THT, TTH\}\end{aligned}$$

- A^c ={not least one heads}

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- A^c ={not least one heads} ={all tails}= $\{TTT\}$
- $P(A^c) =$

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  $$= \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$$

- A^c ={not least one heads} ={all tails}= $\{TTT\}$
- $P(A^c)$ =$1/8$

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- A^c ={not least one heads} ={all tails}= $\{TTT\}$
- $P(A^c)$ =$1/8$
- $P(A) = 1 - P(A^c) = 1 - 1/8 = 7/8$