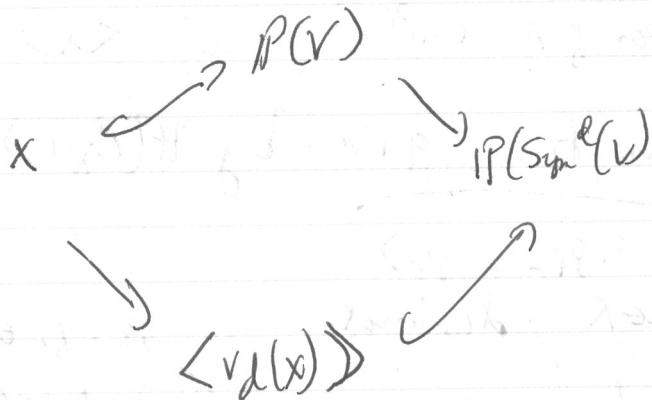


Basic Results: - Joint w Buczyński, Landsberg

$X \subset \mathbb{P}(V)$  smooth,  $v_d: \mathbb{P}(V) \rightarrow \mathbb{P}(\text{Sym}^d(V))$



always  $\sigma_2(v_d(X)) \subset \langle v_d(X) \rangle \cap \sigma_2(v_d(\mathbb{P}^n))$

1) Have equality of  $d \geq r-1 + \text{codim}(X)$

2) So eqns defining  $\sigma_2(v_d(X))$  are "inherited" from eqns defining  $\sigma_2(v_d(\mathbb{P}^n))$

which are the determinantal eqns.

Key Lemma: All as above &  $R$  a degree  $= r$  zero dim sub scheme, then  $v_d(R \cap X) = v_d(R) \cap v_d(X)$

Uniqueness:  $p \in \text{Sym}^a \mathbb{P}(\text{Sym}^d(V))$  if  $r_{v_d(\mathbb{P}^n)}(p) \leq \frac{d+1}{2}$

then  $\text{br}(p) = \sigma(p)$ .

3) Fat All fails of  $X$  is singular.

- (1) Basic Notation  $X \subset \mathbb{P}(V)$  a variety over  $\mathbb{C}$
- (2)  $\langle X \rangle =$  linear span of  $X \subset \mathbb{P}(V)$  i.e.  $\langle X \rangle$  is the proj space w/ lin fns given by  $H^0(\mathcal{O}_X(V))$ .
- (3)  $\sigma_r(X) = \bigcup_{\substack{p \in X \\ \text{distinct}}} \langle p_1, \dots, p_r \rangle$   $p_i, q_i \in X$
- (4) If  $p \in \mathbb{P}(V) \notin \langle p_1, \dots, p_r \rangle$  but  $\exists q_1, \dots, q_{r-1}$  s.t.  $p \in \langle q_1, \dots, q_{r-1} \rangle$  then  $r_X(p) = r$
- If  $p \in \sigma_r(X) \setminus \sigma_{r-1}(X)$  then  $br_X(p) = r$
- note  $r_X \leq br_X$  always.
- (5) Rmk: rank more intuitive, but  $br$  more computable
- (6) Goal: Find eqns for  $X$  &  $\sigma_r(X) \subset \mathbb{P}(V)$
- (7) Impossible as stated in sense anything is possible. equations are not intrinsic to  $X$
- (8)  $v_d: \mathbb{P}(V) \rightarrow \mathbb{P}(\text{Sym}^d(V))$ : Abbreviated  $\mathbb{P}^n \rightarrow \mathbb{P}^N$   $N = \binom{n+d}{d} - 1$

Cont Fix a basis of  $F_1, \dots, F_N \in \text{Sym}^d(V^V)$  then

$$v \in V \mapsto F_1(v), \dots, F_N(v)$$

Don't want to take: hyper planes  $H \subset \mathbb{P}^N \iff$

$$V(F) \subset \mathbb{P}^n \text{ is hyperplanes } \mathbb{P}^N \iff \text{hypersurfaces of deg} = d$$

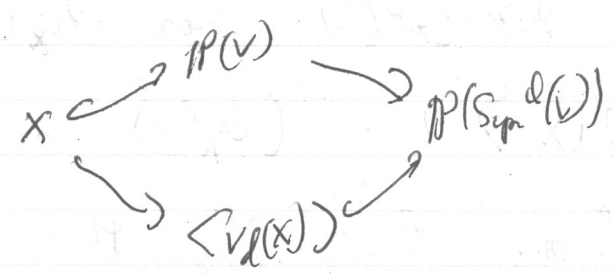
so  $X \subset V(F) \iff v_d(X) \subset H$  a hyper plane

$v_d(\mathbb{P}^n) \subset \mathbb{P}^N$  is defined by quadratic eqns

From this (Mumford)  $v_d(X)$  is defined by quadratic eqns in  $\langle v_d(X) \rangle$

Generalize to  ~~$\sigma_r(X)$~~ : argued to  $\sigma_n(X)$

Picture



So Always  $\sigma_n(v_d(X)) \subset \sigma_n(v_d(\mathbb{P}^n)) \cap \langle v_d(X) \rangle$

so have 'determinantal eqns'

18) What are eqns of  $\sigma_r(v_d(P^n))$  (are they eqns)

19) Complete eqns in  $\sigma_r(v_d(P^n))$  not known

20) Hope they are in degree  $r+1$  : Buczyński & Buczyńska show the obvious ones are not enough. These are the det eqns though.

21) But do we have equality  $\sigma_r(v_d(X)) = \langle v_d(X) \rangle \cap \sigma_r(v_d(P^n))$ ?

22) Our result: true set theoretically once  $d \geq \text{Cat}(X) + r - 1$

23) What is  $\text{Cat}(X)$ : smallest integer so that have C-M. regularity of  $\mathcal{O}_X$

24) If  $d \geq \text{Cat}(X)$  then  $h_X(d) = h^0(\mathcal{O}_X(d)) = \langle v_d(X) \rangle^{-1}$  ( $\mathcal{O}_X(d) \cong \mathcal{O}_X(1) \otimes v_d$ )

25) Recall our thm: Man th:

$d \geq r-1 + \text{Cat}(X)$  then  $\sigma_r \hookrightarrow$

$$\sigma_r(v_d(X)) = \langle v_d(X) \rangle \cap \sigma_r(v_d(P^n)).$$

26) Follows from key lemma & proving that  
 a subscheme that is smoothable on  $\mathbb{P}^n$  is smoothable  
 on  $X \rightarrow$  This uses  $X$  smooth

27) Key lemma  $\langle v_d(R \wedge X) \rangle = \langle v_d(K) \rangle \wedge \langle v_d(R) \rangle$   
 ( $R_f$  is local alg of Hilbert polys)

28) Small rank thm of  $r(p) \leq \frac{d+1}{2} \Rightarrow$   
 $r(p) = br(p)$  also follows. In essence  
 lemma  $\Rightarrow$  uniqueness.

29) Results fail to give  $\text{sing } X$  -  
 $\sigma_1(v_d(X)) \not\subseteq \sigma_1(\mathbb{P}^n) \langle v_d(X) \rangle +$   
 Need eqns of large degree.