1. Shumway and Stoffer, Exercise 3.2. In addition, answer the following question:
(f) Obviously, the answer to (e) is “yes” (why else would they be asking this question if it weren’t?). Suppose you have already simulated \( x_1, \ldots, x_{1000} \) using the proposed model and \( x_1 \) as in (e). How could you simulate \( x_0 \) so that \( x_0, x_1, \ldots, x_{1000} \) has the distribution of a sequence of length 1001 from a stationary, mean 0 AR(1) process? Show that your proposed method yields a process with the desired mean and autocovariance function.

2. Consider the AR(2) model \((1 - \frac{1}{2}B^2)x_t = w_t\).
   (a) Write \( x_t \) as a sum of the form \( \sum_{j=0}^{\infty} \alpha_j w_{t-j} \) and use your result to derive an explicit expression for the autocorrelation function for \( \{x_t\} \).
   (b) If you wanted to simulate a mean 0, stationary series of length 1000 from this model, how could you do it? In particular, how would you get the series started (i.e., how would you simulate \( x_1 \) and \( x_2 \)?)

3. For \( n = 2^{14} + j \) with \( j = 0, \pm 1, \pm 2, \pm 3, \pm 4 \), give the command in R \( \text{system.time(fft(1:n))} \). How do your results for the “user time” relate to the prime factorizations of each of these values of \( n \)? Note that, especially for the faster computations, the value of “user time” may vary noticeably if you repeat the command. (If you are not running R on a unix or linux machine, I’m not positive this command will work, so try this out early to make sure you can get the results.)

4. Periodograms.
   (a) Mimicking what I did in class for the Los Angeles mortality data, make plots of the periodogram of the first differences of the monthly CO\(_2\) levels from Mauna Loa both including and excluding the value for March 2008.
   (b) How do the results compare? Which periodogram is more helpful? Explain.
   (c) Make a third periodogram plot in which you append a 0 to the end of the series of 599 first differences obtained when excluding March 2008. How does this plot compare to the other two?