
Homework 5. Due Wed November 10.

1. **A Barrier Option.** Use the Optional Stopping Theorem to solve problem 2 from the midterm: A contract pays the owner 1 share of stock $S$ at the first time $t = 1, 2, ..., T$ that the share price $S_t$ exceeds $X$. If the share price does not exceed $X$ at any time $t = 0, ..., T$, the contract pays one share of stock $S$ at time $T$. Find the price of this contract.

2. **Another Barrier Option.** We considered in the previous homework a barrier option which pays $(S_n - K)^+$ for $K = 105$ if $S_n$ has first crossed the barrier $X = 95$. We now allow the owner of the option to exercise early. Find the price premium for the right to early exercise.

3. **An American Put.** It the following problem, use the following values. You can use either R or Splus.

```r
n<-10
r<-.log(1.05)/n
u<-.1.10
d<-.90
ut <- exp(-r)*u
dt <- exp(-r)*d
piH <- (1-dt)/(ut -dt)
piT <- (ut-1)/(ut -dt)
S0 <- 100
K<-95
```

We are interested in the intrinsic value process $G_t = (K - S_t)^+$, $0 \leq t \leq n$. Let $\tau$ be the optimal exercise time.

(a) Find the distribution of the random variable $E[G_{\tau} | \tau]$ (both a table and a bar graph would be good).

(b) Deduce from (a) the value of the American put with strike price $K$.

Read Chapter 4 in Shreve I, and do as many problems as you have time for. Also start reading Chapter 1-2 in Shreve II.