It the following problems, use the same values as in the lecture. You can use either R or Splus.

\[ M \leftarrow 1000 \]
\[ n \leftarrow 100 \]
\[ r \leftarrow \log(1.05)/n \]
\[ u \leftarrow 1.01 \]
\[ d \leftarrow 0.99 \]
\[ u_t \leftarrow \exp(-r) \cdot u \]
\[ d_t \leftarrow \exp(-r) \cdot d \]
\[ \pi_H \leftarrow (1-d_t)/(u_t -d) \]
\[ \pi_T \leftarrow (u_t-1)/(u_t -d) \]
\[ S_0 \leftarrow 100 \]

You may wish to check out the R commands “source” and “function” for these problems.

1. **Barrier option.** We wish to determine the price \( v \) of a barrier options which pays \( E(S_n - K)^+ \) for \( K = 105 \) if \( S_n \) has first crossed the barrier \( X = 95 \).

   (a) Estimate by simulation the value \( v \).

   (b) Find, by theoretical means, a Markov process \( Z_t \) so that \( Z_t \) is a two component vector and so that the value \( V_t \) of the option at time \( t \) can be written \( V_t = f(Z_t) \).

   (c) Use the result in (b) to calculate \( v \) exactly in R. You need to build a tree. – For this specific subproblem, you may use Matlab if you prefer.

2. **Optimal sampling.** We are interested in estimating the price call option price \( E(S_n - K)^+ \) for \( K = 110 \). To this end, for each value of \( Q(H) = .45, .5, .55, .6, .65, .7 \), do the following. Create \( R = 100 \) replications of your estimated price (since each price is based on \( M=1000 \) drawings, you are here drawing \( M \cdot R = 100,000 \) binomial random variables). Use the “var” command in R to determine the variability of your estimate.

   Create a plot that gives the variances for each \( Q(H) \). On the basis or your results, explain which value of \( Q(H) \) you would prefer to use.