In the following problems, all markets are assumed to be arbitrage–free, and to contain a riskless asset (called BOND), with riskless rate of return \( r \).

1. **Numeraire Invariance.** Prove Proposition 1 in the notes.

2. **Put–Call Parity.** Consider a single–period market with a risky asset \( \text{STOCK} \), a \( \text{CALL} \) option with strike \( K \), and a \( \text{PUT} \) option with strike \( K \). Assume that the rate of return on the riskless asset BOND is \( r = 0 \).
   (a) Show that there is a replicating portfolio in the assets \( \text{STOCK}, \text{CALL}, \text{and BOND} \) for the asset PUT.
   (b) Deduce a formula for the \( t = 0 \) market price of PUT in terms of the \( t = 0 \) market prices of \( \text{STOCK} \) and \( \text{CALL} \).
   (c) Explain how your answers to parts (a) and (b) must be modified if \( r > 0 \).

3. **Stocks with Dividends.** Consider a \( T \)--period market with a riskless asset BOND whose rate of return is \( r = 0 \). Let \( \text{STOCK} \) be a risky asset that, at each \( t = 1, 2, \ldots, T \), pays a dividend of \( \delta S_t \) shares of BOND, where \( 1 > \delta > 0 \) is a fixed constant and \( S_t \) is the current share price of \( \text{STOCK} \). Let \( \pi \) be an equilibrium distribution for the market.
   (a) Show that
   \[
   S_0 = (1 + \delta)^T \sum_{\omega \in \Omega} \pi(\omega) S_T(\omega),
   \]
   where the sum is over all possible market scenarios \( \omega \).
   (b) What is the forward price \( F_0 \) of asset \( \text{STOCK} \)? (NOTE: The forward contract is an agreement made at \( t = 0 \) for a BUYER to pay \( F_0 \) shares of BOND at \( t = T \) in exchange for one share of \( \text{STOCK} \).)

4. Let \( \mathcal{M} \) be a homogeneous, \( T \)--period binary market with a risky asset \( \text{STOCK} \) whose share price follows equations (26)-(27) of the notes. Assume that the market \( \mathcal{M} \) has a riskless asset BOND with rate of return \( r = 0 \). Consider a contract FLOOR that pays the BUYER one share of BOND at every time \( t = 1, 2, \ldots, T \) when the share price of \( \text{STOCK} \) is below its initial value \( S_0 \). What is the arbitrage price of one FLOOR at time \( t = 0 \)?