Problem 1. Suppose \( X \) has density \( f(x) = c/x^6 \) for \( x > 1 \) and \( f(x) = 0 \) otherwise, where \( c \) is a constant.
(a) Find \( c \).
(b) Compute \( \mathbb{E}X \).
(c) Compute \( \text{Var}X \).

Problem 2. (a) Suppose \( X \) is a uniform random variable on \((0, 1]\). Find the density function of the random variable \( Y = -\log X \).
(b) Suppose that \( Y \) has is a continuous random variable with c.d.f. \( F(x) = P\{Y \leq x\} \). What is the distribution of \( F(Y) \)?

Problem 3. Let \( Z = (X_1, X_2, X_3) \) be a random point, written in rectangular coordinates \( X_1, X_2, X_3 \), chosen from the uniform distribution on the interior of the unit ball in 3 dimensions.
(a) What is the probability density of the distance \( R = \sqrt{X_1^2 + X_2^2 + X_3^2} \)?
(b) What is the probability density of the first coordinate \( X_1 \)?

Problem 4. Let \( X \) be a random variable with density \( f(x) = ce^{-x^2-x} \), where \( c \) is a constant.
(a) Find \( c \).
(b) Compute \( \mathbb{E}X \).