Assumption: Let $X$ be a zero mean weakly stationary process with absolutely summable $k$th order cumulant functions $c_k$,

$$\sum_{u_1,\ldots,u_{k-1} \in \mathbb{Z}} |c_k(u_1, \ldots, u_{k-1})| < \infty.$$

Definition of the periodogram:

$$d^{(T)}(\lambda) = \sum_{t=1}^{T} h^{(T)}(t) X(t) \exp(-i\lambda t)$$

$$I^{(T)}(\lambda) = \frac{1}{2\pi H^{(T)}(0)} |d^{(T)}(\lambda)|^2$$

Properties:

- The mean of the periodogram is given by

$$\mathbb{E}(I^{(T)}(\lambda)) = \int_{\Pi} \frac{|H^{(T)}(\mu)|^2}{2\pi H^{(T)}_2(0)} f(\lambda - \mu) d\mu.$$  

The periodogram is asymptotically unbiased.

- The asymptotic variance of the periodogram is

$$\lim_{T \to \infty} \text{var}(I^{(T)}(\lambda)) = f(\lambda)^2,$$

that is, the periodogram is not consistent.

- The periodogram ordinates are asymptotically independent and $\chi^2$-distributed.
Leakage effect and tapering

Leakage effect:

- Transfer of power from one region to another
- caused by large sidelobes in spectral window $\Phi_2^{(T)}$

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Leakage effect and tapering

Types of biases
- Local bias: due to central lobe, determines resolution of estimate
- Broad-band bias: due to sidelobes (leakage effect)

Tradeoff between local and broad-band bias:

Note: There is also a tradeoff between bias (both types) and variance of nonparametric (periodogram based) spectral estimates.
Kernel spectral estimation

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