Stat 317/253 Winter 2014 HW #18 February 26

Due **Monday, March 5**, **in class** (at the beginning of the lecture period)

Readings: Section 8.2.2, 8.5.1

Problems for Self-Study: Exercise 8.24 (Solutions in p.764)

Problems to Turn In:

1. (M/G/1) Recall in class we have shown that there is a discrete-time Markov chain $\{Y_n, n \geq 0\}$ embedded in the M/G/1 queueing model. Let

$$\begin{aligned} Y_0 &= 0 \\ Y_n &= \# \text{ of customers in the system} \\ &\quad\ \text{leaving behind at the } n\text{th departure, } n \geq 1 \\ A_n &= \# \text{ of customers entered the system} \\ &\quad\ \text{during the service time of the } n\text{th customer, } n \geq 1 \end{aligned}$$

Observed that $\{Y_n, n \geq 0\}$ and $\{A_n, n \geq 1\}$ are related as follows

$$Y_{n+1} = A_{n+1} + (Y_n - 1)^+ = \begin{cases} Y_n - 1 + A_{n+1} & \text{if } Y_n > 0 \\ A_{n+1} & \text{if } Y_n = 0 \end{cases}$$

Recall in HW7 we have shown that the mean of the stationary distribution $\lim_{n\to\infty} \mathbb{E}[Y_n]$ is

$$\mathbb{E}[A_n] + \frac{\mathbb{E}[A_n(A_n - 1)]}{2(1 - \mathbb{E}[A_n])}$$

Verify that $\lim_{n\to\infty} \mathbb{E}[Y_n]$ is equal to

$$\lambda \mathbb{E}[S] + \frac{\lambda^2 \mathbb{E}[S^2]}{2(1 - \lambda \mathbb{E}[S])}.$$

2. Exercise 8.19 on p.571 in [IPM10e] (Just a continuous-time Markov chain problem.)