1. (M/G/1) Recall that the M/G/1 model is a queueing model assuming
   • Customers arrive in accordance with a Poisson process of rate $\lambda$;
   • i.i.d service times with a general distribution $G$, $S_i \sim G$;
   • a single server; and
   • first come, first serve

   In class we have shown that there is a discrete-time Markov chain $\{Y_n, n \geq 0\}$ embedded in
   the M/G/1 queueing model, in which
   
   \[ Y_0 = 0 \]
   \[ Y_n = \# \text{ of customers in the system} \]
   \[ \text{leaving behind at the } n\text{th departure}, \ n \geq 1 \]
   \[ A_n = \# \text{ of customers entered the system} \]
   \[ \text{during the service time of the } n\text{th customer}, \ n \geq 1 \]

   Observed that $\{Y_n, n \geq 0\}$ and $\{A_n, n \geq 1\}$ are related as follows

   \[ Y_{n+1} = A_{n+1} + (Y_n - 1)^+ = \begin{cases} Y_n - 1 + A_{n+1} & \text{if } Y_n > 0 \\ A_{n+1} & \text{if } Y_n = 0 \end{cases} \]

   In class we have shown that $A_n$’s are i.i.d. with distribution

   \[ \alpha_k = P(A_n = k) = \int_0^\infty P(A_n = k|S_n = y)G(dy) = \int_0^\infty \frac{(\lambda y)^k}{k!} e^{-\lambda y}G(dy) \]

   Recall in HW7 we have derived the generating function of the stationary distribution of such
   a process

   \[ G(s) = E[s^{Y_n}] = (1 - g'(1)) \frac{(s - 1)g(s)}{s - g(s)} \]

   in which $g(s) = E[s^{A_n}] = \sum_{k=0}^{\infty} \alpha_k s^k$ is the generating function of $A_n$’s Show that the mean of
   the stationary distribution $\lim_{n \to \infty} E[X_n]$ is

   \[ G'(1) = g'(1) + \frac{g''(1)}{2(1 - g'(1))} = E[A_n] + \frac{E[A_n(A_n - 1)]}{2(1 - E[A_n])} = \lambda E[S] + \frac{\lambda^2 E[S^2]}{2(1 - \lambda E[S])}. \]

   You might want to use the L’Hôpital’s Rule

2. Exercise 8.19 on p.571 in [IPM10e] (Just a continuous-time Markov chain problem.)