STAT22000 Autumn 2013 Lecture 14&15

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A Remark on the Notation for Means and Variances

The **mean**, or **expected value**, or **expectation** of a random variable X can be denoted as

- μ_X
- $\mu(X)$
- $\mathbb{E}(X)$ (Here "$\mathbb{E}$" means "expectation")

The **variance** of a random variable X can be denoted as

- σ_X^2
- $\sigma^2(X)$
- $\mathrm{Var}(X)$

Variances of Random Variables

Recall that for random variables X and Y,

- $\mathbb{E}(X+Y) = \mathbb{E}(X) + \mathbb{E}(Y)$ (always valid)
- $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ when X and Y are independent

Question: What about $\mathrm{Var}(X-Y)$?

In general, if $X_1, X_2, \ldots, X_n$ are random variables, then

- $\mathbb{E}(X_1 + X_2 + \cdots + X_n) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \cdots + \mathbb{E}(X_n)$
  - This is always valid no matter X_i's are independent or not
- $\mathrm{Var}(X_1+X_2+\cdots+X_n) = \mathrm{Var}(X_1)+\mathrm{Var}(X_2)+\cdots+\mathrm{Var}(X_n)$ when $X_1, X_2, \ldots, X_n$ are **independent.**

Four Rolls of a Die (1)

The two properties on the previous slide are very useful since you can find the mean and variance for $X_1 + X_2 + \cdots + X_n$ without knowing the distribution of $X_1 + X_2 + \cdots + X_n$.

Example: What is the mean and variance for the sum of the number of spots one gets when rolling a die 4 times?

Approach 1

- Let S_4 be the total number of spots in 4 rolls.
- Possible values of S: 4, 5, 6, . . . , 23, 24
- Distribution of S_R?
  - e.g., $P(S_4 = 15) = ?$
    How many ways are there to have a sum of 15 in 4 rolls?
  - $6^4 = 1296$ possible outcomes, too many to enumerate
- Is there an easier way?

Four Rolls of a Die — Approach 2

- Let X_1, X_2, X_3, and X_4 be respectively the number of spots in the 1st, 2nd, 3rd, and 4th roll.
- Observe that $S_4 = X_1 + X_2 + X_3 + X_4$
- X_1, X_2, X_3, and X_4 have a common distribution:

value	1	2	3	4	5	6
probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

- mean: $\mathbb{E}(X_1) = 1\cdot\frac{1}{6} + 2\cdot\frac{1}{6} + 3\cdot\frac{1}{6} + 4\cdot\frac{1}{6} + 5\cdot\frac{1}{6} + 6\cdot\frac{1}{6} = 3.5$
- $\mathrm{Var}(X_1) = 1^2\cdot\frac{1}{6}+2^2\cdot\frac{1}{6}+3^2\cdot\frac{1}{6}+4^2\cdot\frac{1}{6}+5^2\cdot\frac{1}{6}+6^2\cdot\frac{1}{6}-\mathbb{E}(X_1)^2 = \frac{35}{12}$
- X_2, X_3, and X_4 have the same mean and variance as X_1 since they have a common distribution
- So $\mathbb{E}(S_4) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \mathbb{E}(X_4)$
  $= 3.5 + 3.5 + 3.5 + 3.5 = 14.$
- Since X_1, X_2, X_3, and X_4 are independent, we have
  $\mathrm{Var}(S_4) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \mathrm{Var}(X_3) + \mathrm{Var}(X_4)$
  $= \frac{35}{12} + \frac{35}{12} + \frac{35}{12} + \frac{35}{12} = \frac{35}{3}.$

Many Rolls of a Die

The second approach can be easily generalized to more rolls. Consider the total number of spots S_n got in n rolls of a die, and let X_i be the number of spots got in the ith roll, for $i = 1, 2, \ldots, n$. Then

$$S_n = X_1 + X_2 + \cdots + X_n$$

and all the X_i's have a common distribution with mean 3.5 and variance 35/6. The mean and variance of S_n are hence

$$\mathbb{E}(S_n) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \cdots + \mathbb{E}(X_n) = 3.5 \times n$$
$$\mathrm{Var}(S_n) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_n) = \frac{35}{12} \times n$$

since X_i's are independent of each other.
The mean and variance S_n can be found without first working out the distribution of S_n.

Sum and Mean of i.i.d. Random Variables

The rolling die example demonstrates a common scenario for many problems: suppose $X_1, X_2, \ldots, X_n$ are **i.i.d.** random variables with mean μ and variance σ^2.

- Here, "**i.i.d.**" = "**independent**, and **identically distributed**", which means that $X_1, X_2, \ldots, X_n$ are independent and have identical probability distributions.

The mean and variance of $S_n = X_1 + X_2 + \cdots + X_n$ are then

$$\mathbb{E}(S_n) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \cdots + \mathbb{E}(X_n) = \mu \times n = n\mu$$
$$\mathrm{Var}(S_n) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_n) = \sigma^2 \times n = n\sigma^2$$

- Observe $\mathrm{Var}(S_n) = n\sigma^2 \geq \mathrm{Var}(X_i) = \sigma^2$, the sum of X_i's has greater variability than a single X_i does.

What if Not Independent?

In general, if X and Y are NOT independent, then

$$\mathrm{Var}(X+Y) = \mathrm{Var}(Y) + \mathrm{Var}(X) + 2\rho\sigma(X)\sigma(Y).$$

Here, ρ is the **correlation** between X and Y, which is defined analogously to the **(sample) correlation** r.

$$\text{sample correlation } r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$
$$\text{correlation } \rho = \mathbb{E}\left[\left(\frac{X - \mu_X}{\sigma_X}\right)\left(\frac{Y - \mu_Y}{\sigma_Y}\right)\right]$$

- We'll NEVER compute ρ in STAT220. The formula is FYI only.

Correlation between two independent variables is zero.

Properties of Correlation ρ

Let ρ be the correlation of random variables X and Y. ρ has very similar properties with the sample correlation r.

- $-1 \leq \rho \leq 1$
- If X and Y are independent, then $\rho = 0$
  (But when $\rho = 0$, X and Y may not be independent.)
- If $\rho > 0$ then when X gets big, Y also tends to gets big, and vice versa. In this case,
  $$\mathrm{Var}(X+Y) > \mathrm{Var}(Y) + \mathrm{Var}(X).$$
- If $\rho < 0$ then when X increases, Y tends to decrease, and vice versa. In this case,
  $$\mathrm{Var}(X+Y) < \mathrm{Var}(Y) + \mathrm{Var}(X).$$
- If $\rho = 1$ or -1, then there exists constants a and b such that Y always equals $aX + b$.

A Statistical Model of Simple Random Sampling

Consider a population comprised of N individual, indexed by $1, 2, 3, \ldots, N$. Each individual has a numerical characteristic such that x_i is the numerical characteristic of the ith individual.

Example. The population is the 50,000 people age 25 and over in this town, indexed from $1, 2, 3, \ldots, N = 50{,}000$. Let x_i be the years of schooling of the ith individual in the population.

When a *single* individual is selected *at random* from the population (everyone has $1/N$ chance to be selected), how many years of schooling X did he/she get?

- X is a random variable
- What is the probability distribution of X?

$$p_x = P(X = x) = \frac{\#\text{ of people who have got } x \text{ years of schooling}}{N}$$

Suppose the frequency table of $x_1, \ldots, x_{50000}$ is

years of schooling x	count	proportion p_x
0	500	0.01
1	500	0.01
2	500	0.01
3	500	0.01
4	500	0.01
5	1000	0.02
6	1000	0.02
7	1000	0.02
8	3000	0.06
9	2000	0.04
10	2000	0.04
11	2000	0.04
12	17000	0.34
13	3000	0.06
14	3000	0.06
15	3000	0.06
16	9500	0.19
Total	50000	1

The table of years x v.s. proportion p_x is exactly the probability distribution of a single draw X.

Then mean of X is

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i = E(X) = \sum_x x p_x$$
$$= 0 \times 0.01 + 1 \times 0.01 + \cdots + 16 \times 0.19$$
$$\approx 11.8$$

and the variance of X is

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 = \mathrm{Var}(X)$$
$$= \sum_x (x-\mu)^2 p_x$$
$$= (0-11.8)^2 \times 0.01 + \cdots + (16-11.8)^2 \times 0.19$$
$$\approx 12.96$$

Review of Simple Random Samples

Suppose $X_1, X_2, \ldots, X_n$ are n draws at random **without** replacement from a population of size N. That is,

1. In the first draw, everyone has $1/N$ chance to be selected
2. In the second draw, each of the remaining $N-1$ has $1/(N-1)$ chance to be selected
3. $\vdots$
4. In the nth draw, each of the remaining $N-n+1$ has $1/(N-n+1)$ chance to be selected

Then $\{X_1, X_2, \ldots, X_n\}$ is called a **simple random sample (SRS)** of size n.

Properties of Simple Random Samples

1. Every X_i has **the same probability distribution** (the population distribution X)
2. The X_i's are **(nearly) independent**
   - Since we usually sample **without** replacement, draws are not independent.
   - As long as the sample size n is small ($<10\%$ relative to the population size N, the dependencies among sampled values are small and are generally ignored.
   - When sampling from an infinite population ($N=\infty$), the X_i's are independent.

Due to the reasons above, we often assume observations X_1, $X_2, \ldots, X_n$ in a simple random sample are **i.i.d.** from some (population) distribution.

Mean and Variance of Sample Means

In sampling and many other cases, the **population mean** μ is often *unknown*. The **sample mean** $\overline{X}_n = (X_1 + \cdots + X_n)/n$ is often used to estimate it.

- How good is this estimation?

Observe that $\overline{X}_n = S_n/n$, in which S_n is the sum of $X_1, X_2, \ldots,$ and X_n. Recall we have shown in the beginning that

$$\mathbb{E}(S_n) = n\mu, \quad \text{and} \quad \mathrm{Var}(S_n) = n\sigma^2.$$

By the scaling properties of the expected values and the variances, $\mathbb{E}(cX) = c\mathbb{E}(X)$ and $\mathrm{Var}(cX) = c^2\mathrm{Var}(X)$, we have

$$\mathbb{E}(\overline{X}_n) = \mathbb{E}\left(\frac{1}{n}S_n\right) = \frac{1}{n}\mathbb{E}(S_n) = \frac{1}{n}\cdot n\mu = \mu,$$

$$\mathrm{Var}(\overline{X}_n) = \mathrm{Var}\left(\frac{1}{n}S_n\right) = \left(\frac{1}{n}\right)^2 \mathrm{Var}(S_n) = \frac{1}{n^2}\cdot n\sigma^2 = \frac{\sigma^2}{n}.$$

Properties of the Sample Mean

So far we have shown that: the sample mean $\overline{X}_n$ of i.i.d random variables with mean μ and variance σ^2 has the following properties:

1. $\mathbb{E}(\overline{X}_n) = \mu$. . . . . $\overline{X}_n$ is an **unbiased** estimator for μ.
2. $\mathrm{Var}(\overline{X}_n) = \sigma^2/n$ . . . . The larger n is, the less variable $\overline{X}_n$ is.
3. **Weak Law of Large Numbers:** As n gets large
$$\overline{X}_n \longrightarrow \mu.$$
   Intuitively, this is clear from the mean and the variance of $\overline{X}_n$; the "center" of the distribution $\overline{X}_n$ is μ, and the "spread" around it becomes smaller and smaller as n grows.
4. The distribution of $\overline{X}_n$, called the **sampling distribution** of the sample mean, depends on the distribution of X_i.
   - hard to find in general, except for a few cases
   - When n is large, we have **Central Limit Theorem**!

Central Limit Theorem (CLT)

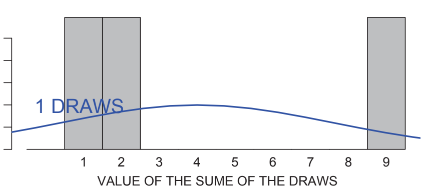
Let $X_1, X_2, \ldots$ be a sequence of **i.i.d.** random variables (discrete or continuous) with **mean** μ **and variance** σ^2. Then, when n is large,

- the distribution of the sample mean $\overline{X}_n = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$ is approximately
$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right).$$
- the distribution of the sum $X_1 + X_2 + \cdots + X_n$ is approximately
$$N(n\mu, \sqrt{n}\sigma).$$

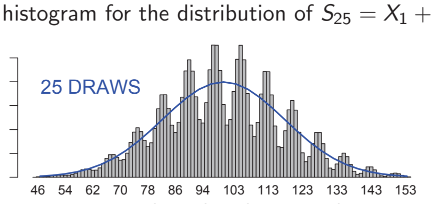
If X_i's are i.i.d., with the distribution

value	1	2	9
probability	1/3	1/3	1/3

Probability histogram for the distribution of X_1:



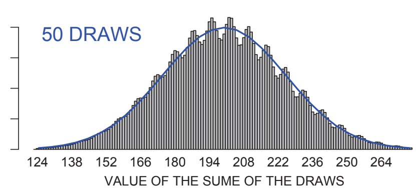
Probability histogram for the distribution of $S_{25} = X_1 + \cdots + X_{25}$:



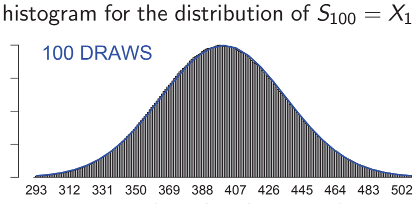
X_i's are i.i.d., with the distribution

value	1	2	9
probability	1/3	1/3	1/3

Probability histogram for the distribution of $S_{50} = X_1 + \cdots + X_{50}$:



Probability histogram for the distribution of $S_{100} = X_1 + \cdots + X_{100}$:



Example: For the years of schooling example, it is known that the population distribution has mean $\mu = 11.8$ and variance is $\sigma^2 = 12.96$. For a sample of size 400, by CLT, the sample mean $\overline{X}_n$ is approximately

$$N\Big(11.8, \sqrt{\frac{12.96}{400}}\Big) = N(11.8, 0.18).$$

- Find the probability that the sample mean < 11.
- Find the probability that the sample mean is between 11.8 ± 0.36.

Example: Shipping Packages

Suppose a company ships packages that vary in weight:

- Packages have mean 15 lb and standard deviation 10 lb.
- Packages weights are independent from each other

Q: What is the probability that 100 packages will have a total weight exceeding 1700 lb?

Let W_i be the weight of the ith package and

$$T = \sum_{i=1}^{100} W_i, \qquad \mu_T = 100\mu_W = 100(15) = 1500\text{lb}$$

$$\sigma_W^2 = 100\sigma_W^2 = 100(10^2), \qquad \sigma_W = \sqrt{100(10)^2} = 100\,\text{lb}.$$

By CLT, T is approximately $N(1500, 100)$, and 1700 is 2SD above the mean, so the probability is about 2.5%.

Summary: Means and Sums of i.i.d. Random Variables

Suppose $X_1, X_2, \ldots, X_n$ are *i.i.d.* random variables with mean μ and variance σ^2.

Let $S_n = X_1 + X_2 + \cdots + X_n$ and $\overline{X}_n = S_n/n$ be respectively the **sum** and the **sample mean** of $X_1, X_2, \ldots, X_n$.

So far we have shown that S_n and $\overline{X}_n$ have the following properties

	sum S_n	sample mean $\overline{X}_n$
expected value	$\mathbb{E}(S_n) = n\mu$	$\mathbb{E}(\overline{X}_n) = \mu$.
variance	$\mathrm{Var}(S_n) = n\sigma^2$	$\mathrm{Var}(\overline{X}_n) = \sigma^2/n$
sampling distribution for small n	no general form	no general form
approximate sampling distribution for large n	$N(n\mu, \sqrt{n}\sigma)$	$N\Big(\mu, \frac{\sigma}{\sqrt{n}}\Big)$

Bernoulli Random Variables (1)

A random variable X is said to a **Bernoulli** random variable if it takes two values only: 0 and 1.

- $p = P(X = 1)$ is called the **probability of success**
- Then $P(X = 0)$ must be $1 - p$ since X is either 0 or 1.
- So the distribution of a Bernoulli random variable with probability p of success must be

value of X	0	1
probability	$1-p$	p

- Mean and variance:

$$\begin{aligned}
\mathbb{E}(X) &= 0 \cdot (1-p) + 1 \cdot p = p, \\
\mathrm{Var}(X) &= 0^2 \cdot P(X=0) + 1^2 \cdot P(X=1) - \mathbb{E}(X)^2 \\
&= 0 \cdot (1-p) + 1 \cdot p - p^2 = p(1-p)
\end{aligned}$$

Bernoulli Random Variables (2)

Bernoulli distribution arises when a random phenomenon has only two possible outcomes, e.g.,

- heads or tails in one coin tossing: $X = 1$ if heads, $X = 0$ if tails
- success or failure in a trial: $X = 1$ if success, $X = 0$ if failure
- whether a product is defected: $X = 1$ if defected, $X = 0$ if not
- whether a person uses iPhone: $X = 1$ if yes, $X = 0$ if no

Binomial Distribution (1)

A random variable Y is said to have a **Binomial** distribution $B(n, p)$, denoted as $Y \sim B(n, p)$, if it is a **sum of *n* i.i.d. Bernoulli** random variables, $X_1, X_2, \ldots, X_n$, with probability p of success.

Binomial distribution arises when we count the number of "successes" in a series of n *independent* "trials", e.g.,

- number of heads when tossing a coin n times ("success" = heads)
- # of defected items in a batch of size 1000 ("success" = defected)
- # of iPhone users in a SRS from a huge population ("success" = iPhone user)

Mean and Variance of Binomial

Recall a Binomial random variable $Y \sim B(n, p)$ are sums of i.i.d. Bernoulli random variables $X_1, X_2, \ldots, X_n$, with probability p of success. The mean and variance of Y are thus

$$\begin{aligned}
\mathbb{E}(Y) &= \mathbb{E}(X_1) + \mathbb{E}(X_2) + \cdots + \mathbb{E}(X_n) \\
&= p + p + \cdots + p = np \\
\mathrm{Var}(Y) &= \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_n) \\
&= p(1-p) + p(1-p) + \cdots + p(1-p) = np(1-p)
\end{aligned}$$

since X_i's are i.i.d. with mean p and variance $p(1-p)$.

What about the distribution of Y? E.g., What is $P(Y = 3)$?

Factorials and Binomial Coefficients

The notation $n!$, read **n factorial**, is defined as

$$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$$

e.g.,

$$1! = 1, \qquad 3! = 1 \times 2 \times 3 = 6,$$
$$2! = 1 \times 2 = 2, \qquad 4! = 1 \times 2 \times 3 \times 4 = 24.$$

By convention, $0! = 1$.

The **binomial coefficient**: $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

- which is the number of ways to choose k items, regardless of order, from a total of n distinct items
- $\binom{n}{k}$ is read as "n choose k".

e.g.,

$$\binom{4}{2} = \frac{4!}{2! \times 2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6, \quad \binom{4}{4} = \frac{4!}{4! \times 0!} = \frac{4!}{4! \times 1} = 1$$

Binomial Formula

The distribution of a Binomial distribution $B(n, p)$ is given by the **binomial formula**. If Y has the binomial distribution $B(n, p)$ with n trials and probability p of success per trial, the probability to have k successes in n trials, $P(Y = k)$, is given as

$$P(Y = k) = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for } k = 0, 1, 2, \ldots, n.$$

Why the binomial formula is true?
See the next slide for an example.

Why is the Binomial Formula True? (Optional)

Let Y be the number of success in 4 independent trials, each with probability p of success. So $Y \sim B(4, p)$.

- To get 2 successes ($Y = 2$), there are 6 possible ways:

  SSFF SFSF SFFS FSSF FSFS FFSS

  in which "SSFF" means success in the first two trials, but not in the last two, and so on.
- As trials are independent, by the multiplication rule,

$$\begin{aligned}
P(\text{SSFF}) &= P(\text{S})P(\text{S})P(\text{F})P(\text{F}) \\
&= p \cdot p \cdot (1-p) \cdot (1-p) = p^2(1-p)^2 \\
P(\text{SFSF}) &= P(\text{S})P(\text{F})P(\text{S})P(\text{F}) \\
&= p \cdot (1-p) \cdot p \cdot (1-p) = p^2(1-p)^2
\end{aligned}$$

- Observe all 6 ways occur with probability $p^2(1-p^2)$, because all have 2 successes and 2 failures

So $P(Y = 2) = (\#\text{ of ways}) \times (\text{prob. of each way}) = 6 \cdot p^2(1-p)^2$

Why is the Binomial Formula True? (Optional)

In general, for $Y \sim B(n, p)$

$$\begin{aligned}
P(Y = k) &= (\text{Number of ways to have exactly } k \text{ success}) \\
&\quad \times P(\text{success in all the first } k \text{ trials} \\
&\qquad\quad \text{and none of the last } n-k \text{ trials}) \\
&= (\text{Number of ways to choose } k \text{ out of } n) \times p^k(1-p)^{n-k} \\
&= \binom{n}{k} p^k (1-p)^{n-k}
\end{aligned}$$

Example

Four fair dice are rolled simultaneously, what is the chance to get (a) exactly 2 aces? (b) exactly 3 aces? (c) 2 or 3 aces?

- A trial is one roll of a die. A success is to get an ace.
- Probability of success $p = 1/6$
- number of trials $n = 4$ is fixed in advance
- Are the trials independent? Yes!
- So $Y = \#$ of aces got has a $B(4, 1/6)$ distribution

(a) $P(Y = 2) = \dfrac{4!}{2!\,2!} \left(\dfrac{1}{6}\right)^2 \left(1 - \dfrac{1}{6}\right)^2 = \dfrac{25}{216}$

(b) $P(Y = 3) = \dfrac{4!}{3!\,1!} \left(\dfrac{1}{6}\right)^3 \left(1 - \dfrac{1}{6}\right)^1 = \dfrac{5}{324}$

(c) $P(Y = 2 \text{ or } Y = 3) = P(Y = 2) + P(Y = 3)$
$= \dfrac{25}{216} + \dfrac{5}{324} = 0.131$

Requirements to be Binomial (1)

To be a Binomial random variable, check the following

1. the number of trials n must be fixed in advance,
2. p must be identical for all trials
3. trials must be independent

Q1: A SRS of 50 from all UC undergrads are asked whether or not he/she is usually irritable in the morning. X is the number who reply yes. Is X binomial?

- a trial: a randomly selected student reply yes or not
- prob. of success $p =$ proportion of UC undergrads saying yes
- number of trials $= 50$
- Strictly speaking, NOT binomial, because trials are not independent
- Since the sample size 50 is only 1% of the population size (≈ 5000), trials are nearly independent
- So X is approximately binomial, $B(n = 50, p)$

Requirements to be Binomial (2)

Q2 John tosses a fair coin until a head appears. X is the count of the number of tosses that John makes. Is X binomial?

- one trial = one toss of the coin
- number of trials is not fixed
- NOT binomial

Q3 Most calls made at random by sample surveys don't succeed in talking with a live person. Of calls to New York City, only 1/12 succeed. A survey calls 500 randomly selected numbers in New York City. X is the number that reach a live person. Is X binomial?

- one trial = a call that reach a live person
- number of trials $n = 500$
- probability of success $p = 1/12$
- Independent trials? Huge population, so (nearly) independent
- $X \sim B(500, 1/12)$

CLT for Counts and Proportion

Let $X_1, X_2, \ldots$ be a sequence of **i.i.d.** Bernoulli random variables with **probability** p **of success**. So X_i has mean $\mu = p$ and variance $\sigma^2 = p(1-p)$. Then

- The sum $S_n = X_1 + X_2 + \cdots + X_n$ now is the **count** of X_i's that take value "1", and has a binomial distribution $B(n, p)$. As n gets large, the distribution of S_n is approximately
$$N(n\mu, \sqrt{n}\sigma) = N(np, \sqrt{np(1-p)}).$$
- The sample mean $\overline{X}_n = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$ is just the **proportion** of X_i's that take value "1." As n gets large, the distribution of $\overline{X}_n$ is approximately
$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) = N\left(p, \frac{\sqrt{p(1-p)}}{\sqrt{n}}\right).$$

Example: Twitter Users

Suppose 20% of the internet users use Twitters. If a SRS of 2500 internet users are surveyed, what is the probability that the percentage of Twitter users in the sample is over 21%?