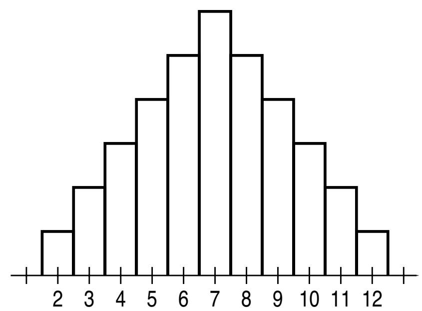
STAT22000, Aut 2013, Solutions to HW5 Self-Study Problems

4.57. **(a)** The pairs are given below. We must assume that we can distinguish between, for example, "(1,2)" and "(2,1)"; otherwise, the outcomes are not equally likely. **(b)** Each pair has probability $1/36$. **(c)** The value of X is given below each pair. For the distribution (given below), we see (for example) that there are four pairs that add to 5, so $P(X = 5) = \frac{4}{36}$. Histogram below, right. **(d)** $P(7 \text{ or } 11) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$. **(e)** $P(\text{not } 7) = 1 - \frac{6}{36} = \frac{5}{6}$.

(1,1) 2	(1,2) 3	(1,3) 4	(1,4) 5	(1,5) 6	(1,6) 7
(2,1) 3	(2,2) 4	(2,3) 5	(2,4) 6	(2,5) 7	(2,6) 8
(3,1) 4	(3,2) 5	(3,3) 6	(3,4) 7	(3,5) 8	(3,6) 9
(4,1) 5	(4,2) 6	(4,3) 7	(4,4) 8	(4,5) 9	(4,6) 10
(5,1) 6	(5,2) 7	(5,3) 8	(5,4) 9	(5,5) 10	(5,6) 11
(6,1) 7	(6,2) 8	(6,3) 9	(6,4) 10	(6,5) 11	(6,6) 12



Value of X	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

4.59. The table on the right shows the additional columns to add to the table given in the solution to Exercise 4.57. There are 48 possible (equally-likely) combinations.

(1,7) 8	(1,8) 9
(2,7) 9	(2,8) 10
(3,7) 10	(3,8) 11
(4,7) 11	(4,8) 12
(5,7) 12	(5,8) 13
(6,7) 13	(6,8) 14

Value of X	2	3	4	5	6	7	8	9	10	11	12	13	14
Probability	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{6}{48}$	$\frac{6}{48}$	$\frac{6}{48}$	$\frac{5}{48}$	$\frac{4}{48}$	$\frac{3}{48}$	$\frac{2}{48}$	$\frac{1}{48}$

5.19. **(a)** $\mu_{\bar{x}} = 0.5$ and $\sigma_{\bar{x}} = \sigma/\sqrt{50} = 0.7/\sqrt{50} \doteq 0.09899$. **(b)** Because this distribution is only approximately Normal, it would be quite reasonable to use the 68–95–99.7 rule to give a rough estimate: 0.6 is about one standard deviation above the mean, so the probability should be about 0.16 (half of the 32% that falls outside ± 1 standard deviation). Alternatively, $P(\bar{x} > 0.6) \doteq P\left(Z > \frac{0.6 - 0.5}{0.09899}\right) = P(Z > 1.01) = 0.1562$.

5.25. If W is total weight, and $\bar{x} = W/25$, then:

$$P(W > 5200) = P(\bar{x} > 208) \doteq P\left(Z > \frac{208-190}{5/\sqrt{25}}\right) = P(Z > 2.57) = 0.0051$$

5.26. **(a)** Although the probability of having to pay for a total loss for one or more of the 12 policies is very small, if this were to happen, it would be financially disastrous. On the other hand, for thousands of policies, the law of large numbers says that the average claim on many policies will be close to the mean, so the insurance company can be assured that the premiums they collect will (almost certainly) cover the claims. **(b)** The central limit theorem says that, in spite of the skewness of the population distribution, the average loss among 10,000 policies will be approximately Normally distributed with mean \$250 and standard deviation $\sigma/\sqrt{10{,}000} = \$1000/100 = \$10$. Since \$275 is 2.5 standard deviations above the mean, the probability of seeing an average loss over \$275 is about 0.0062.