

STAT22000, Autumn 2013 Homework 5

All page, section, and exercise numbers below are for the course text (Moore, McCabe and Craig, Introduction to the Practice of Statistics, 7th edition).

Reading: Section 4.3, 4.4, 5.1

Problems for Self-Study: (Do Not Turn In. Solutions are at the end of the textbook.)

1. Exercise 4.57 on p.258
2. Exercise 4.59 on p.258
3. Exercise 5.19 on p.310
4. Exercise 5.25 on p.311
5. Exercise 5.26 on p.311

Problems to Turn In: due Wednesday, Nov. 6, in class

- 10:30 Session: Do #1, #3, #4, #5, and choose one between #2 and #6
- 1:30 Session: Do #1, #2, #3, #4, #5 only

1. Revision of **Exercise 4.58 on p.258**

Nonstandard dice can produce interesting distributions of outcomes. You have two balanced 6-sided dice. One is a standard die, with faces having 1, 2, 3, 4, 5, 6 spots. The other die has three faces with 0 spots and three faces with 6 spots.

- (a) Write down all $2 \times 6 = 12$ possible pairs of faces.
- (b) When rolling the two dice, let X be the number of spot(s) on the up-face of the standard die, Y be the number of spot(s) on the up-face of the non-standard die, and T be the total number of spots on the two dice. For each of the outcomes in (a), write down the values of X , Y , and T for that outcome.
- (c) Use the result of (b) to give the probability distributions of X , Y , and T .
- (d) Are X and Y independent? (You can argue by the nature of rolling dice, not by the definition of independence of random variables.)
- (e) Find the means μ_X , μ_Y , μ_T of X , Y , and T , and verify that $\mu_T = \mu_X + \mu_Y$.
- (f) Find the variances σ_X^2 , σ_Y^2 , σ_T^2 of X , Y , and T and verify that $\sigma_T^2 = \sigma_X^2 + \sigma_Y^2$.

2. (**Optional for 10:30 session, Required for 1:30 session**)

Revision of **Exercise 4.50 and 4.51 on p. 257**

Suppose that 20% of the internet users use Twitter to post updates about themselves or to see updates about others. Think about selecting simple random samples (SRSs) from the population of all internet users. You can assume that the population of all internet users is so huge that drawing a few users from it won't change the proportion of Twitter users in it.

Sam is going to choose a simple random sample of size 2. Let S_2 be number of Twitter users in Sam's sample. Peter also plans to choose another simple random sample of size 3 (different from Sam's sample). Let S_3 be number of Twitter users in Peter's sample.

- (a) What are the possible values of S_2 and S_3 respectively?

- (b) Look at the two people in Sam's sample in order. The 4 possible outcomes that Sam might get is

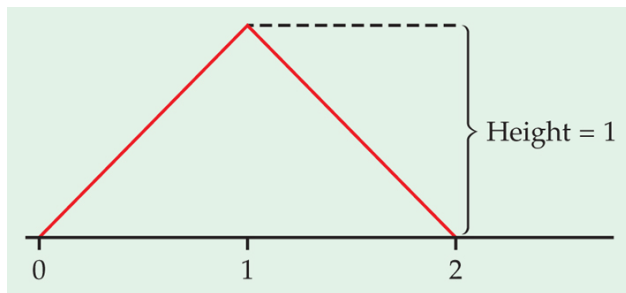
TT TN NT NN.

Here T means a Twitter user and N means a non-Twitter users. For example, TT means both people are Twitter users, TN means the first one is a Twitter user but the second is not, and so on. Find the probability for each of the four outcomes.

- (c) What is the value of S_2 for each outcome in (b)? Use the result in (b) to find the distribution of S_2 .
- (d) Using similar argument as in (b), list all 8 possible outcomes for Peter's sample and find the probability for each of them.
- (e) What is the value of S_3 for each outcome in (d)? Use the result in (d) to find the distribution of S_3 .
- (f) Find the mean and the variance for S_2 .
- (g) Find the mean and the variance for S_3 .
- (h) One can use the proportion of twitter users in the sample as an estimator of the population proportion $p = 0.2$. The sample proportion is $\hat{p}_2 = S_2/2$ in Sam's sample, and $\hat{p}_3 = S_3/3$ in Peter's sample. Show that the mean of \hat{p}_2 and \hat{p}_3 are both $p = 0.2$. (This shows that both \hat{p}_2 and \hat{p}_3 are unbiased estimator for p .)
- (i) Find the variances for \hat{p}_2 and \hat{p}_3 . Which one has smaller variance?

3. Exercise 4.64 on p.259 — The sum of two uniform random numbers.

Generate *two* random numbers between 0 and 1 and take Y to be their sum. Then Y is a continuous random variable that can take any value between 0 and 2. The density curve of Y is the triangle shown in Figure 4.12.



- (a) Verify by geometry that the area under this curve is 1.
- (b) What is the probability that Y is less than 1? (Sketch the density curve, shade the area that represents the probability, then find that area. Do this for (c) also.)
- (c) What is the probability that Y is **more** than 0.6?

4. Exercise 5.22 on p.310 — A lottery payoff. A \$1 bet in a state lottery's Pick 3 game pays \$500 if the three-digit number you choose exactly matches the winning number, which is drawn at random. Here is the distribution of the payoff X :

Payoff X	\$0	\$500
Probability	0.999	0.001

Each day's drawing is independent of other drawings.

- (a) What are the mean and standard deviation of X ?
 - (b) Joe buys a Pick 3 ticket twice a week. What does the law of large numbers say about the average payoff Joe receives from his bets?
 - (c) What does the central limit theorem say about the distribution of Joe's average payoff after 104 bets in a year?
 - (d) Joe comes out ahead for the year if his average payoff is greater than \$1 (the amount he spent each day on a ticket). What is the probability that Joe ends the year ahead?
5. **Exercise 5.24 on p.311 — Flaws in carpets.** The number of flaws per square yard in a type of carpet material varies with mean 1.3 flaws per square yard and standard deviation 1.5 flaws per square yard. This population distribution cannot be Normal, because a count takes only whole-number values. An inspector studies 200 square yards of the material, records the number of flaws found in each square yard, and calculates \bar{x} , the mean number of flaws per square yard inspected. Use the central limit theorem to find the approximate probability that the mean number of flaws exceeds 2 per square yard.

6. **For 10:30am session only**

Suppose the joint distribution of X and Y is given by

$$f_{X,Y}(x, y) = 3(x^2y + xy^2) \quad \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1$$

and 0 elsewhere.

- (a) Verify that $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x, y) dx dy = 1$
- (b) Find the marginal density $f_X(x)$ of X , and the marginal density $f_Y(y)$ of Y .
- (c) Find the conditional density $f_{Y|X}(y|x)$ of Y given X .
- (d) Are X and Y independent?
- (e) Find $E(X)$.