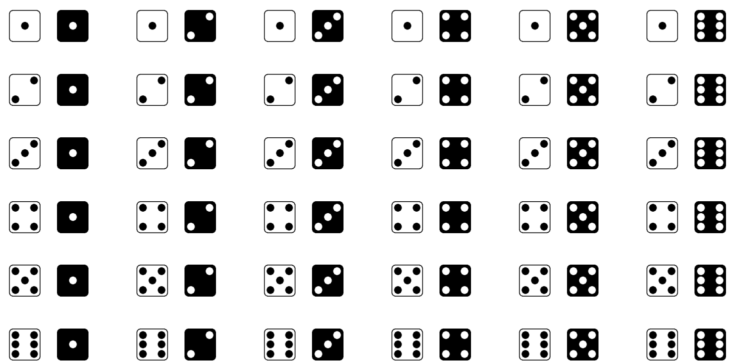
LISTING THE WAYS

• A pair of dice are to be thrown. What is the chance of getting a total of 7 spots?

- There are _______ possible ways for 2 dice to fall:

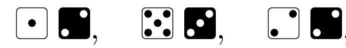


- By symmetry, all these ways are _______ _______ , so each one has chance _______ to happen.
- There are _______ ways to get a total of 7 spots:

- The chance of getting a total of 7 spots equals _______ .

• When figuring chances, one helpful strategy is to write down a complete list of all the possible ways that the chance process can turn out.

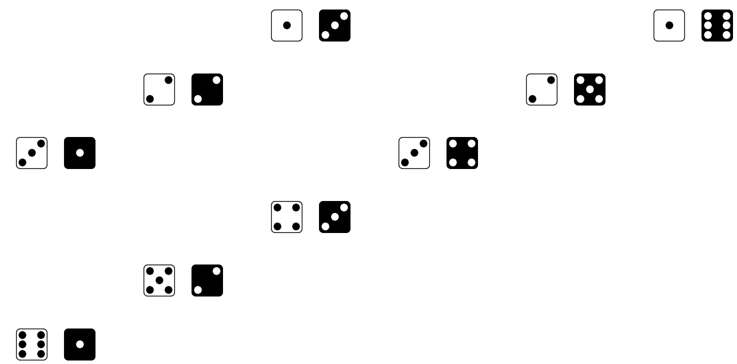
• Consider the following game. You get to throw a pair of dice repeatedly. You win if you roll a total of 4 spots before a total of 7 spots. For example, if you roll



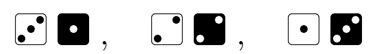
then you win. But if you roll

then you lose. What's the chance of your winning?

- The rolls that terminate play are:



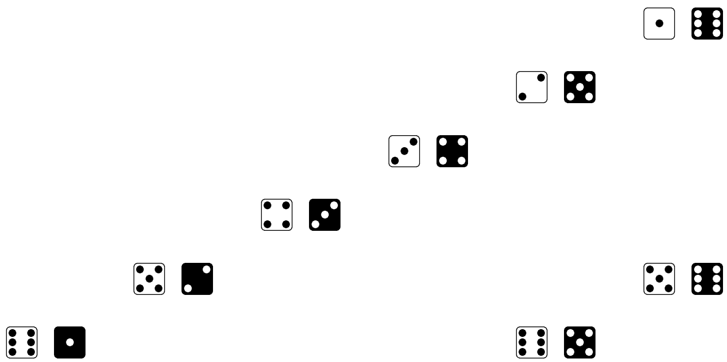
- By symmetry, all these ways are _______ _______ , so each one has chance _______ to happen.
- There are _______ ways to win:



- The chance of winning equals _______ .

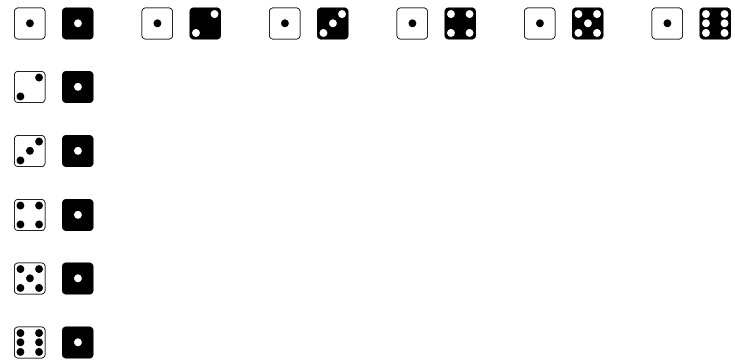
THE ADDITION RULE

• A pair of dice are to be thrown. What is the chance of getting a total of 7 spots, or of 11 spots?



- The chance equals ________ .
- Is this the chance of getting a total of 7 spots, plus the chance of getting a total of 11 spots?
  - The chance of getting a total of 7 spots equals ________ .
  - The chance of getting a total of 11 spots equals ________ .
  - So, the chance for 7 or 11 ________ the chance for 7, plus the chance for 11.

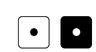
• Two dice — one white, one black — are to be thrown. What is the chance of getting at least one ace?



- The chance equals ________ .
- Is this the chance that the white die lands ace, plus the chance that the black die lands ace?
  - The chance that the white die lands ace is ________ .
  - The chance that the black die lands ace is ________ .
  - So, the chance for at least one ace does ____________ the chance of an ace on the white die, plus the chance of an ace on the black die.

• Why does blindly adding the individual chances give the wrong answer?

- Adding the individual chances ____________ the chance that both dice land aces:



- When are two things mutually exclusive?

  - Two things are mutually exclusive when the occurrence of one prevents the occurrence of the other.

  - Suppose a white die and a black die are to be thrown. The following outcomes ________ mutually exclusive:
    - Getting a total of 7.
    - Getting a total of 12.

  - But the following outcomes ________ mutually exclusive:
    - Getting an ace on the white die.
    - Getting an ace on the black die.

- What is the *addition rule* for two mutually exclusive events?

  - If two things are mutually exclusive, then the chance that at least one of those things will happen equals the sum of the individual chances.

  - If the two things aren't mutually exclusive, adding the individual chances will give the wrong answer, due to double-counting.

- Is there an addition rule for three or more mutually exclusive events?

  - ________ . The chance that at least one of several things will happen equals the sum of the individual chances, provided that the occurrence of any one of the things prevents the occurrence of each of the other ones.

"INDEPENDENT" VERSUS "MUTUALLY EXCLUSIVE"

- Two things are *independent* if the conditional chances for the second one given the first are the same, no matter how the first one turns out.

- Two things are *mutually exclusive* when the occurrence of one prevents the occurrence of the other.

- Two tickets will be drawn at random *with* replacement from the box

1	2	3	4

  - Consider these two events:
    - Event A: The first ticket drawn is [4].
    - Event B: The second ticket drawn is [4].
  - True or false: events A and B are independent.
    - This is ________ .
  - True or false: events A and B are mutually exclusive.
    - This is ________ .

- As above, but for drawing *without* replacement:
  - True or false: events A and B are independent.
    - This is ________ .
      - Given that A happens, the conditional chance for B equals ________ .
      - Given that A doesn't happen, the conditional chance for B equals ________ .

- True or false: events A and B are mutually exclusive.
  - This is ________ .

- Whether or not two things are independent, or mutually exclusive, depends not only on the things themselves, but also on the chance process involved!

WHEN TO ADD, AND WHEN TO MULTIPLY?

- Try to visualize the chance process that the problem is about — throwing dice, dealing cards, or whatever it is.
- Identify the event whose chance is asked for.
- Try to connect this event to simpler things whose chances you know.
- You may want to compute the chance that at least one of these simpler things will happen.
  - In that case, add the chances of the simpler things, provided they are ______________ .
- Or, you may want to compute the chance that all of the simpler things will happen.
  - In that case, multiply the unconditional chances of the simpler things, provided they are ______________ .
  - If the simpler things are not independent, you need to need to use the more complicated "multiplication rule," which involves ______________ probabilities.
- True or false: If you see the words "mutually exclusive," add the chances. If you see the word "independent," multiply the chances.
  - This is _______. You first have to think about what chance you need to find.
- Solving a complicated problem may involve several steps, some using the addition rule, some using the multiplication rule, and some using the complementation rule.

THE BIRTHDAY PROBLEM

- What is the chance that among _______ people chosen at random, at least two of them have a common birthday?
  - We can estimate this using a chance model:
    - The chance process is like making _______ draws at random _______ replacement from a box containing tickets numbered from 1 to 365.
    - We want to find the chance that some ticket is drawn more than once.
  - The chance that all the draws are distinct can be reasoned out as follows:
    - The first draw can be anything.
    - The second draw could be any of _______ tickets, _______ of which are different from the first draw.
    - The third draw could be any of _______ tickets, _______ of which are different from the first two draws.
    - And so on. By the __________ rule, the chance that all _______ draws are different equals

$$\frac{\_\_}{365} \times \frac{\_\_}{365} \times \cdots \times \frac{\_\_}{365} = \_\_\_\_\_\_\_ .$$

  - The chance of at least one matching pair equals _______ .
- This problem was solved by first setting up a chance model, then using the multiplication rule to find the chance of the opposite thing (all birthdays distinct), and finally using the complementation rule to find the chance of the thing itself (a common birthday).

• The following table gives the chance p_k of drawing some ticket more than once in the course of k draws with replacement from a box with tickets numbered from 1 to 365.

k	p_k	k	p_k
1	0.0000	22	0.4757
2	0.0027	23	0.5073
3	0.0082	24	0.5383
4	0.0164	25	0.5687
5	0.0271	26	0.5982
6	0.0405	27	0.6269
7	0.0562	28	0.6545
8	0.0743	29	0.6810
9	0.0946	30	0.7063
10	0.1169	31	0.7305
11	0.1411	32	0.7533
12	0.1670	33	0.7750
13	0.1944	34	0.7953
14	0.2231	35	0.8144
15	0.2529	36	0.8322
16	0.2836	37	0.8487
17	0.3150	38	0.8641
18	0.3469	39	0.8782
19	0.3791	40	0.8912
20	0.4114	41	0.9032
21	0.4437		

POKER

• In poker, what's the chance of being dealt a full house (one pair and three of a kind)?

• Think about where the two cards that give you the pair lie among the 5 cards you're dealt. The possibilities are:

1	P	P	T	T	T
2	P	T	P	T	T
3	P	T	T	P	T
4	P	T	T	T	P
5	T	P	P	T	T
6	T	P	T	P	T
7	T	P	T	T	P
8	T	T	P	P	T
9	T	T	P	T	P
10	T	T	T	P	P

• Are these possibilities mutually exclusive?

• ________ . So we can get the chance of getting a full house by ________ the individual chances.

• The first possibility has chance $\frac{52}{52} \times \frac{3}{51} \times \frac{48}{50} \times \frac{3}{49} \times \frac{2}{48}$, or 0.000144 .

• The last possibility has chance $\frac{}{52} \times \frac{}{51} \times \frac{}{50} \times \frac{}{49} \times \frac{}{48}$, or ________ .

• Each of the 10 possibilities has chance ________ .

• The chance of being dealt a full-house is 10×0.000114, or 0.00114.

• This problem was solved by first listing the ways, then using the multiplication rule to find the chance of each way, and finally using the addition rule to add the chances.

CRAPS

- The game of *craps* is played with 2 dice. On the first roll:
  - You lose (crap out) if you get a total of 2, 3, or 12.
  - You win if you get a total of 7 or 11.
  - Otherwise, the total becomes your *point*, and you continue to roll the dice:
    - If you get your point before a total of 7, you win.
    - Otherwise, you lose.

- What is the chance of winning at craps?
  - The different ways you can win are:

    A: Roll a total of _______ or _______ to start with.

    B: Roll a total of _______ to start with, go on to win.

    C: Roll a total of _______ to start with, go on to win.

    D: Roll a total of _______ to start with, go on to win.

    E: Roll a total of _______ to start with, go on to win.

    F: Roll a total of _______ to start with, go on to win.

    G: Roll a total of _______ to start with, go on to win.
  - Are these possibilities mutually exclusive?
    - _______ . So we can get the chance of winning by _______ the individual chances.

- The chance of A (7 or 11 to start with) equals _______ .
- What is the chance of B (start with a point of 4, and make it)?
  - Here we want to know the chance that two things will both happen. We can use the _____________ rule:
    - The chance of rolling a total of 4 to start with equals _______ .
    - Given that 4 is your point, the conditional chance to make it equals the chance of rolling a total of 4 before a total of 7.
      - That equals _______ .
  - So the chance of B equals _______ $\times$_______ .

- The chance of the other possibilities (involving points of 5, 6, 8, 9, and 10) can be figured in the same way:
  - Chance of C equals $\frac{4}{36} \times \frac{4}{4+6}$.
  - Chance of D equals $\frac{5}{36} \times \frac{5}{5+6}$.
  - Chance of E equals $\frac{5}{36} \times \frac{5}{5+6}$.
  - Chance of F equals $\frac{4}{36} \times \frac{4}{4+6}$.
  - Chance of G equals $\frac{3}{36} \times \frac{3}{3+6}$.
- We get the overall chance of winning by adding the chances of A, B, . . . , G. That comes to $\frac{244}{495} \approx 0.493$. Thus the chance at winning at craps is slightly under _______%.

SUMMARY

• When figuring chances, one helpful strategy is to write down a complete list of all the possible ways that the chance process can turn out. If this is too hard, at least write down a few typical ways, and count how many ways there are in total.

• The chances that at least one of several things will happen equals the sum of the individual chances, provided that the things are mutually exclusive. Otherwise, adding the chances will give the wrong answer, due to double counting.

• “Independent” and “mutually exclusive” are not synonyms.

• If you are having trouble working out the chance of an outcome, try to figure out the chance of its opposite; then subtract from 100%.

• Solving a complicated problem may involve several steps, some using the addition rule, some using the multiplication rule, and some using the complementation rule.