User Guide for Diagnostic Test

• The Results
  • Are likelihood ratios for the test results presented (or calculatable)?

• Will it help with my patients?
  • Will the reproducibility of the test result and its interpretation be satisfactory in my setting?
  • Are the results applicable to my patients?
  • Will the results change my management?
  • Will patients be better off as a result of the test?
Several equivalent methods

- 2x2 table method
- Decision tree method
- Bayes’s theorem
- The likelihood ratio (odds form)
- Each method produces the same result
General Method

• Start with a current probability (=prior probability, = prevalence) of having the disease $P(D+)$
• Obtain the result of a test $T$
• Revise our estimate of probability to take $T$ into account: $P(D+|T)$
2x2 tables

- Sens = TP/ (TP+FN)
- Spec = TN/(TN+FP)
- FP rate = 1-Spec
- FN rate = 1-Sens
- Prevalence = (TP+FN)/(TP+FN+FP+TN)
- PPV = TP/(TP+FP)
- NPV = TN/(FN+TN)
PPV and NPV from prevalence, sens, spec

- ECG for Dx of acute MI: Sens=0.8, Spec=0.95
- Prevalence: 0.5
- Work backwards from prev & sens to fill in first column
- Use (1-prev) & spec to fill in second column
- Now use rows to calculate PPV and NPV

<table>
<thead>
<tr>
<th></th>
<th>MI+</th>
<th>MI-</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKG+</td>
<td>400</td>
<td>25</td>
<td>425</td>
</tr>
<tr>
<td>EKG-</td>
<td>100</td>
<td>475</td>
<td>575</td>
</tr>
<tr>
<td>Total</td>
<td>500</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

- PPV = $\frac{400}{425} = 0.94$
- NPV = $\frac{475}{575} = 0.83$
Bayes’s Theorem

- Relationship of odds to probability:
  - Odds = \( p/(1-p) \)
  - Prob = odds / (1+odds)

- p = 2/3
- O = 2/1 = 2
Bayes’s Theorem

What are the odds that this patient has the disease?

**Prior Odds**

This will typically depend on the prevalence

\[
\frac{P(D+)}{P(D-)}
\]

Example: MI

Prevalence = 0.5

Prior odds ratio = 0.5 / 0.5 = 1
Bayes’s Theorem

We would like to know the odds that an individual with a positive test actually has the disease. This is called the posterior odds. It is also called the predictive odds with a positive test.

This is based on the probability of having the disease or not, considering only those who would test positive.

\[
\frac{P(D+|T+)}{P(D-|T+)} \quad \text{Prior odds}
\]

\[
\frac{P(D+)}{P(D-)}
\]

This is called the \textit{posterior odds}.

It is also called the \textit{predictive odds with a positive test}. 

Bayes’s Theorem

The posterior odds depends on how accurate the test is

\[
\frac{P(D+|T+)}{P(D-|T+)} = \frac{P(T+|D+)}{P(T+|D-)} \times \frac{P(D+)}{P(D-)}
\]

How do we combine the prior odds with the likelihood ratio to obtain the posterior odds?
Bayes’s Theorem

\[ \frac{P(D+|T+)}{P(D-|T+)} = \frac{P(T+|D+)}{P(T+|D-)} \times \frac{P(D+)}{P(D-)} \]

posterior odds = likelihood ratio \times prior odds

updated info = info from test \times original info
# Updating information

## Bayes’s Theorem

<table>
<thead>
<tr>
<th></th>
<th>CA +</th>
<th>CA -</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRE +</td>
<td>62</td>
<td>127</td>
<td>189</td>
</tr>
<tr>
<td>DRE -</td>
<td>10</td>
<td>101</td>
<td>111</td>
</tr>
<tr>
<td>Totals</td>
<td>72</td>
<td>228</td>
<td>300</td>
</tr>
</tbody>
</table>

- **Post-test Odds**
  - \[
  \frac{62}{127} = \frac{0.86}{(1-0.44)} \times \frac{72}{228}
  \]

- **Likelihood Ratio**
  - \[
  1.54 = \frac{1}{2.0} \times \frac{1}{3.2}
  \]
Example: Diagnosis of acute MI

(Johnson HA, JAMA, 1991; 265: 2229-2231)

- ECG: sens=0.80, spec = 0.95
- Creatine kinase (CK): sens=.95, spec=0.97
- Hx, PE suggest 50-50 chance of MI
  - This gives P(D+)
  - Based on judgment, presentation, sex, etc

- Prior odds of MI = 1
- LR for positive ECG = .80/.05  16.

- Thus, pos ECG → odds = 16:1 in favor of MI
  (posterior probability = 16/(1+16)  0.94)
The next day

- Prior odds of MI = 16 (at this point).
- We get the CK tests back
- CK is positive:
- LR for + CK = 32, thus
- Posterior odds = $16 \times 32 = 500$
- Posterior probability = 0.998
Example: ECG and CK negative

- Prior odds = 1
- LR for negative ECG = .95/.20 = 4.75
- Posterior odds of no MI = 4.75
- Posterior prob of no MI = 4.75/5.75 = 0.826.
  [Posterior probability of MI is still 0.174—worth finding out about the CK]
- LR for negative CK = .97/.05  19. Thus, posterior odds becomes 90, for a probability of 0.989 of not having had an MI
Costs of increasing diagnostic certainty

- Johnson’s framework: compare $P(D^+|T^+)$ to $P(D^+)$, that is
- Consider the amount by which a positive result could change your assessment
- The posterior probability depends on both the test characteristics and prevalence
- Ordering a test = buying diagnostic information.
- If $Dx$ is essentially certain, the relative cost of even very inexpensive tests is unwarranted
Logistic regression...

• ...fits the logit (log odds) as a linear function of the predictor variables

• \( \text{logit} = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \)

• The response variable (Y) is a 0-1 outcome

• One needs special computer programs to do logistic regression
  • GLIM and Stata are examples
GI Bleeding in Critical Care Patients


• Is respiratory failure a risk factor for significant GI bleed?
• Example uses Stata
. blogit Bleed N RespFail

Logit Estimates

| _outcome  | Coef.  | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|-----------|--------|-----------|-------|------|----------------------------|
| RespFail  | 3.23835| .6073632  | 5.333 | 0.000| 2.048425 - 4.429245        |
| _cons     | -6.27539 | .5778899  | -10.859 | 0.000| -7.408033 - -5.142746     |

. blogit, or

| _outcome | Odds Ratio | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|----------|------------|-----------|-------|------|----------------------------|
| RespFail | 25.504     | 15.49019  | 5.333 | 0.000| 7.755678 - 83.8681         |
Multiple risk factors

• Some patients have multiple risk factors.
• What is the (additional) effect of respiratory failure after taking all other possible risk factors into account?
• This is given in the “multiple regression” column of the table: odds that are 15.6 larger.
These are essentially the "Multiple regression" odds ratios in Table 4.
A worksheet

- Baseline odds of GI Bleed
- Respiratory failure
- Coagulopathy
- Product = Odds of GI Bleed

\[
\frac{1}{x} \times 15.6 = \frac{0.001426}{x} \times 15.6
\]
\[
\frac{-}{x} \times 4.3 = \frac{0.02224}{x}
\]

Odds = 1 : 45

* Baseline odds = 1 : 700
A worksheet

- Baseline odds of GI Bleed: 0.001426
- Respiratory failure
- Coagulopathy
- Product = Odds of GI Bleed

\[
\begin{align*}
\text{Baseline odds of GI Bleed} & = 0.001426 \\
\text{Respiratory failure} & = \frac{0.001426 \times 15.6}{1} \\
\text{Coagulopathy} & = \frac{0.001426 \times 4.3}{1} \\
\text{Product} & = 0.00613 \\
\text{Odds} & = 1 : 163
\end{align*}
\]
A worksheet

- Baseline odds of GI Bleed
- Respiratory failure
- Coagulopathy
- Product = Odds of GI Bleed

\[
\begin{align*}
\text{Baseline odds of GI Bleed} & : 0.001426 \\
\text{Respiratory failure} & : 1 \times 15.6 = \\
\text{Coagulopathy} & : 1 \times 4.3 = \\
\text{Product} = \text{Odds of GI Bleed} & : 0.09562 \\
\end{align*}
\]

Odds = 1 : 10
How to use?

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Factor</th>
<th>Dilution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neither Respiratory Failure nor Coagulopathy</td>
<td>0.0014</td>
<td>B = 2 / 1403</td>
<td>1:702</td>
</tr>
<tr>
<td>Respiratory Failure Only</td>
<td>0.0223</td>
<td>B x 15.65</td>
<td>1:45</td>
</tr>
<tr>
<td>Coagulopathy Only</td>
<td>0.0061</td>
<td>B x 4.3</td>
<td>1:163</td>
</tr>
<tr>
<td>Both Respiratory Failure and Coagulopathy</td>
<td>0.0959</td>
<td>B x 15.65 x 4.3</td>
<td>1:10</td>
</tr>
</tbody>
</table>