Network dynamics

1. Introduction
What are networks made of?

- How many types of neurons? How many neurons in each type?
- How are neurons connected (what is the connectivity matrix)?
- What are the external inputs?
- What is(are) the neuron model(s)?
- What is(are) the synapse model(s)?
What are networks made of?

- How many types of neurons? How many neurons in each type?
  - 1, 2, \ldots\ populations
  - \( N \sim 10s, 100s, 1000s, \ldots\ N \to \infty \)
What are networks made of?

- How are neurons connected (what is the connectivity matrix)?
  - Fully connected (all-to-all);
  - Randomly connected (e.g. Erdos-Renyi);
  - Spatial structure;
  - With a structure imposed by learning
What are networks made of?

- What are the external inputs?
  - Constant
  - Stochastic (e.g. independent Poisson processes; independent white noise)
  - Temporally/spatially structured
What are networks made of?

- What is(are) the neuronal model(s)?
  - Binary
  - Spiking (LIF, NLIF, HH-type, etc....)
What are networks made of?

- What is(are) the synapse model(s)?
  - Number (synaptic weight, binary networks)
  - Temporal kernel (spiking networks)
What are the questions?

- **Dynamics**: What are the intrinsic dynamics of networks (*spontaneous activity*, in the absence of structured inputs)?

- **Coding**: What is the effect of external inputs on network dynamics? How do networks encode external inputs?

- **Learning and memory**: How are external inputs learned/memorized?
  - How do external inputs modify network connectivity through synaptic plasticity? How is learning implemented?
  - What is the impact of structuring in the connectivity on network dynamics?

- **Computation**: How do networks perform computations?
Network dynamics, 101

**Binary networks**

- Neurons receive inputs (both from outside and from the network itself)...

\[ I_i(t) = I_{iX}(t) + \sum_j J_{ij} S_j(t) \]

- Neurons decide whether to be active, as a function of those inputs

\[ S_i(t + dt) = \Theta (I_i(t) - T) \]

**Spiking networks**

\[ I_i(t) = I_{iX}(t) + \sum_{j,k} J_{ij}(t - t_j^k) \]

Membrane potential \( V_i(t) \)

\[ \tau_i \frac{dV_i}{dt} = -V_i + I_i(t) \]

Spike emitted whenever \( V_i(t) = V_T \)

After the spike, voltage is reset to \( V_R \).
How to visualize network activity in a network model?

**Binary networks**

- Raster plot: spiking activity of whole network vs time

\[
S_i(t) = 1, 0
\]

**Spiking networks**

\[
S_i(t) = \sum_k \delta(t - t_{i}^k)
\]
• Averaging over time: average firing rates of single neurons

\[ \nu_i = \frac{1}{T} \sum_t S_i(t) dt \]

\[ \nu_i = \frac{1}{T} \int_0^T S_i(t) dt \]

⇒ Distributions of firing rates \( P(\nu) \)
• Averaging over neurons: instantaneous average rate (vs time)

\[ \nu(t) = \frac{1}{N} \sum_i S_i(t) \]  

\Rightarrow \text{Autocorrelation, power spectrum of } \nu
Quantifying firing irregularity

- Distribution of interspike intervals (ISI) $P_i(T)$;
  - Mean ISI $\langle T \rangle$; Firing rate $\nu = 1/T$
  - SD (standard deviation) of ISIs; CV (coefficient of variation) $= \text{SD}/\text{Mean}$
Quantifying correlations

- Cross-correlation function (CCF)
  - Take two spike trains during the interval \([0 : T]\);
  - Choose bin width \(\Delta t\); spike trains become strings of \(T/\Delta t\) binary variables \(S_{i,j}(t)\);
  - For \(\tau = k\Delta t\) compute
    \[
    C_{i,j}(\tau) \propto \frac{1}{T - |\tau|} \sum_{t} S_{i}(t)S_{j}(t + \tau)
    \]

- Spike count correlation (SCC)
  - Choose (long) bin width \(\Delta t\); compute number of spikes in each bin width, \(n_{i,j}(t)\);
  - Compute standard correlation coefficient of spike counts.

- In the limit of long bin widths, \(SCC = \text{time integral of CCF (appropriately normalized)}\).
Visualizing network activity - experimental tools

1. Large-scale imaging methods

- fMRI, EEG, MEG, optical imaging: average activity of thousands of neurons, with a temporal resolution depending on the technique.

<table>
<thead>
<tr>
<th>resolution</th>
<th># neurons</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole cortex</td>
<td>$2 \times 10^{10}$</td>
</tr>
<tr>
<td>1cm</td>
<td>$10^7$</td>
</tr>
<tr>
<td>1mm</td>
<td>$10^5$</td>
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</tbody>
</table>
Visualizing network activity - experimental tools

2. Extracellular recordings

From the raw EFP signal:

- Low-pass filter $\Rightarrow$ ‘local field potential’ (LFP) $\Rightarrow$ presumably reflects average synaptic inputs of $\sim 1000$ cells around the electrode;

- High-pass filter $\Rightarrow$ ‘multi-unit activity’ (MUA) $\Rightarrow$ spike trains of a few neighboring neurons.

- Spike sorting $\Rightarrow$ spike trains of individual neurons (SUA).
Visualizing network activity - experimental tools

3. Multi-unit extracellular recordings

- Allows to record $\sim 100s$ neurons simultaneously
- This is still a very small fraction of neurons in the recorded volume
Visualizing network activity - experimental tools

4. Multi-unit calcium imaging

- Less invasive than electrophysiological recordings
- Temporal resolution is not as good
Visualizing network activity - experimental tools

5. Intracellular recording

⇒ Dynamics of membrane potential $V_i(t)$
⇒ Power spectrum
⇒ Distribution of membrane potentials
⇒ Relationship between mean $V$ and firing rate
Spontaneous activity in cortex

- Neurons fire at low rates (~ 1Hz)
- Broad distribution of firing rates
- Firing is highly irregular - CV ~ 1, similar to a Poisson process
- Large fluctuations in membrane potential
- Correlations are weak to moderate
Distributions of spontaneous firing rates in vivo

Cat visual cortex
Griffith and Horn 1966

Monkey parietal cortex
Koch and Fuster 1989

Mice barrel cortex
O’Connor et al 2010

Rat auditory cortex
Hromadka et al 2008
CVs

Softky and Koch 1993

Compte et al 2003
Correlations

- Average noise correlations range from 0.1-0.4 (Zohary et al. 1994, Bair et al. 2001 ...) to 0.01 (Ecker et al).
- Correlation between noise and signal correlations.
- In most cases, noise correlations do not depend significantly on stimulus (Bair et al. 2001).

Bair et al. 2001 (area MT)
Asynchronous vs synchronous (oscillatory) activity

- Transient oscillatory patterns often seen in LFP recordings (reflects local network activity)

⇒ What are the mechanisms of synchronized oscillations?