STATISTICAL REPORT

STABLE SUBSPACES OVER REGULAR

SOLUTIONS OF MARTINGALE PROBLEMS

by

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Report No. 15 July 1986

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ISSN - 0333-1865

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Abstract.

Let (G_t) be a subfiltration of (F_t) so that the processes $[P(A/G_t)]$ are (F_t)-martingales for all $A \in G_\infty$. Let $\mathrm{stab}_p(G)$ be the smallest stable subspace of H^p which contains the right continuous modifications of these martingales. The paper shows that the martingale $(N_t) \perp \mathrm{stab}_1(G)$ if (N_t) is orthogonal to the martingales $[P(A/G_t)]$. This yields that $\mathrm{stab}_p(G) = \mathrm{stab}_1(G) \cap H^p$ and that the elements of $\mathrm{stab}_1(G)$ remain martingales under any measure $Q \ll P$ so that $Q(A/F_t) = P(A/G_t)$ for all $A \in G_\infty$. The results are applied to regular solutions of stochastic differential equations.

AMS subject classification: 60G44, 60M10, 60M20, 62P20.

Some Keywords: Martingale problems; Regular solutions; Stochastic differential equations; Stable subspaces.

Introduction.

1.1. Some notations, and the main theorem.

Let $(\Omega, F, F_t, P)_{t \geq 0}$ be a filtered probability space with (F_t) right continuous and complete. Let M be the space of right continuous uniformly integrable (F_t, P)-martingales with seminorm $\|(N_t)\|_M = \|N_\infty\|_{L^1}$, and let H^p $(p \in [1,\infty))$ be the set of $(N_t) \in M$ which satisfy $\|(N_t)\|^p_{H^p} \triangleq E(\sup_t |N_t|^p) < \infty$. If $G \subseteq H^p$, let $\mathrm{stab}_p(G)$ be the smallest stable subspcae of H^p which contains G (see Jacod (1979), ch. IV- 1-a, for the definition of stable subspace). If (N_t) and $(L_t) \in M$, then $(N_t) \perp (L_t)$ if $N_0 L_0 = 0$ and $(N_t L_t)$ is a local martingale.

The purpose of this paper is to prove the following result:

Theorem

Let (G_t) be a subfiltration of (F_t), and assume that

$$\forall t \geq 0 \quad \forall A \in G_\infty \quad P(A|G_t) = P(A|F_t) \tag{1.1}$$

Let G be a set of right continuous modifications of the martingales $(P(A|G_t))$ for $A \in G_\infty$ (note that (1.1) ensures that they are adapted to the completion of (G_t)). Then

$$\forall (N_t) \in M \quad (N_t) \perp G \Longrightarrow (N_t) \perp \mathrm{stab}_1(G) \tag{1.2}$$,

$\mathrm{stab}_1(G)$ being the smallest stable subspace over (F_t) which contains G.

1.2. Previous and related results.

The statement (1.2) has previously been proved for other definitions of G. It holds when G is a finite set of martingales and $\mathrm{stab}_1(G) = H^1$, see Jacod (1978), and also Theorem 4.6.7 of Jacod (1979). It also holds when G is a set of continuous martingales. This goes back to Theorem 1.5 (p.88) of Jacod & Yor (1977), see also Proposition 4.13 in Jacod (1979) and Theorem 15 in Strook & Yor (1980). We have not been able to find a formulation where the only condition is that the elements of G be continuous, but this is an easy extension of the mentioned results. Finally, a similar result holds for stable spaces of integrals over random measures, see Yor (1978), and also Theorem 4.53 of Jacod (1979).

The reason why (1.2) is interesting is shown in Jacod (1979). First, it relates the concept of stable subspace of H^p for different values of p. By using the duality between H^p and H^q (for $\frac{1}{p}+\frac{1}{q}=1$, $p\neq 1$, see Theorem 4.7 of Jacod (1979)) together with the Hahn - Banach theorem, (1.2) implies that for all $p\in[1,\infty)$,

$$\mathrm{stab}_p(G) = \mathrm{stab}_1(G)\cap H^p \qquad (1.3)\,.$$

Second, (1.2) connects different extremality-like criteria on P. If F_0 is a 0-1-σ-field, then $\mathrm{stab}_1(G\cup\{1\}) = H^1$ iff P is extremal in the set M of measures under which the elements of G are local martingales, see Theorem 11.2 of Jacod (1979). Using (1.2) (set $N_\infty = \frac{dQ}{dp}$), it is seen that this is equivalent to

$$Q \ll P\,,\ Q\in M \Longrightarrow Q = P \qquad (1.4)\,.$$

A manifestation of this is found in Corollary 11.4 of Jacod (1979).

(1.2) and its consequences can be applied to solutions of martingale problems and to weak solutions of stochastic differential equations. An example of this is Theorem 12.21 of Jacod (1979). In the same way, the result in the present paper can be applied to what is usually called regular weak solutions of stochastic differential equations. E.g., if we consider the equation

$$dX_t = u_t(X)dZ_t , \quad X_0 = x \tag{1.5}$$

where (Z_t) is an (m-dimensional) semimartingale, and u_t is an ($n \times m$-dimensional) predictable functional, then (Ω,F,F_t,P,X_t,Z_t) is a regular weak solution of (1.5) if (1) $\mu(A) = P(Z \in A)$ coincides with a predetermined measue on the function space where Z takes values, (2) (X_t) and (Z_t) satisfy (1.5), and (3)

$$\forall t \geq 0 \quad \forall A \in F_\infty^Z \quad P(A|F_t) = P(A|F_t^Z) \tag{1.6}$$.

For a further discussion of the equation (1.5) and the concept of regular solution, see Jacod (1980), Jacod & Memin (1981) and Lebedev (1983).

An extremal regular weak solution (Ω,F,F_t,P,X_t,Z_t) is one where it holds that if there are measures Q_1 and Q_2 on $F_\infty^{X,Z}$ so that $P = a_1Q_1 + a_2Q_2$ $(a_1,a_2 > 0)$ on $F_\infty^{X,Z}$, and so that $(\Omega,F_\infty^{X,Z},F_t^{X,Z},Q_i,X_t,Z_t)$ is a regular weak solution, then $Q_1 = Q_2 = P$ on $F_\infty^{X,Z}$. If we assume that F_0 is complete and that H is the set of right continuous modifications of the martingales $(P(A|F_t^{X,Y}))$ for $A \in F_\infty^{X,Z}$, then by setting $G_t = F_t^Z$, (1.2) yields for an extremal regular solution that, for any $p \geq 1$,

$$\mathrm{stab}_p(G) = \mathrm{stab}_p(H) \tag{1.7}$$

stab_p being the smallest stable subspace over (F_t). This is because it holds when stab_p is the smallest stable subspace over $(F_t^{X,Z})$. An interesting consequence of this is that

$$\forall A \in F_\infty^{X,Z} \quad \forall t \geq 0 \quad P(A|F_t^{X,Z}) = P(A|F_t) \tag{1.8}$$,

which links extremality with Granger-causality and with the concept of adapted distribution. For this, see sect.4 of Mykland (1986), which also discusses the relationship between Granger-causality and weak uniqueness of a regular solution.

Finally, note that since (1.4) holds for extremal regular weak solutions, there is, for each extremal regular solution (Ω,F,F_t,P,X_t,Z_t) of (1.5), a measurable set A in the (function) space where (X,Z) takes values, so that (Ω,F,F_t,P,X_t,Z_t) is a solution of

$$(1.5) \quad \& \quad (X,Z)(\omega) \in A \qquad (1.9),$$

and so that (1.9) has a weakly unique regular solution. This is easily seen from the fact that the set of measures over the space where (X,Z) takes values is separable. "Weakly unique" here means that for every regular solution (Ω,F,F_t,P,X_t,Z_t) of (1.9), there is no measure Q on $F_\infty^{X,Z}$, $Q \neq P$ on $F_\infty^{X,Z}$, so that $(\Omega,F_\infty^{X,Z},F_t^{X,Z},Q,X_t,Z_t)$ is a regular solution of (1.9).

2. Proof of the theorem.

2.1. Preliminary remarks.

In the following we shall mean the completion of (G_t) when we speak of (G_t). Set

$$I = \mathrm{stab}_1(G)$$

and let $(N_t) \in M$, $(N_t) \perp G$. To show that $(N_t) \perp I$, we can assume without loss of generality that

$$N_t = N_{t\wedge\tau} \quad ,$$

where τ is the time of the first jump by (N_t) (or ∞ if $N_t(\omega)$ does not jump). This is because a general (N_t) can be decomposed into $(N_{t\wedge\tau_1})$, $(N_{t\wedge\tau_2} - N_{t\wedge\tau_1})$, $(N_{t\wedge\tau_3} - N_{t\wedge\tau_2})$, and so on, where τ_i is the i'th jump of (N_t). (N_t) is orthogonal to a martingale if and only if each of these components so is.

Decompose, following Theorem 2.21 (p.32) in Jacod (1979), (N_t) into a continuous and a purely discontinuous part,

$$N_t = N_t^c + N_t^d \qquad .$$

Every $(K_t) \in G$ can be decomposed over (G_t) in the same way as (K_t^c) and (K_t^d), and these are also, respectively, continuous and purely discontinuous martingales over (F_t) as all (G_t)-martingales are (F_t)-martingales. Since $(N_t) \perp G$ it must be orthogonal to (K_t^c), hence $(N_t^c) \perp (K_t^c)$ since $(N_t^d) \perp (K_t^c)$ by Corollary 2.29 (p.37) in Jacod. By the same result, $(N_t^c) \perp (K_t^d)$, hence $(N_t^c) \perp (K_t)$. As $(N_t^c) \perp G$ and is locally bounded, it is by Theorem 4.7 (p.116) $\perp I$.

In the following, we shall write τ_E for the restriction of τ to a set E, i.e. $\tau_E = \tau$ on E and ∞ otherwise. By Theorem 5.16 (p.42) In Elliot (1982), we can write

$$\tau = \tau_A \wedge \tau_B \qquad ,$$

where A and B are disjoint, $A \cup B = \Omega$, $A, B \in F_{\tau-}$, and so that τ_A is accessible and τ_B totally inaccessible.

Consider the set

$$\varepsilon = \left\{ \bigcup_n \{\tau_B = \sigma_n < \infty\} \mid \sigma_n \text{ is a } (G_t)\text{-stopping time} \right\} \quad ,$$

and set

$$\lambda = \sup\{P(E) : E \in \varepsilon\} \quad .$$

Let C_n be a sequence in ε so that $P(C_n) \to \lambda$. As ε is closed under countable union,

$$C = \bigcup_n C_n \in \varepsilon$$

and

$$P(C) = \lambda \quad .$$

Obviously,

$$C \subseteq B$$

and

$$C \in F_{\tau_B} \quad ,$$

hence $\tau_C = \tau_{C \wedge B}$ is a stopping time. We also consider the stopping time τ_D, where

$$D = B - C \in F_{\tau_B} \quad .$$

As (N_t^c) is orthogonal to I, it remains to show that the same result holds for the pure jump martingale (N_t^d). We shall begin this by studying the jumps in A, C and D separately. From this, (N_t^d) will naturally be decomposed

$$N_t = L_t^{(1)} + L_t^{(2)} \quad ,$$

$(L_t^{(1)})$ taking care of the jump over D and $(L_t^{(2)})$ of the rest of the jump. We show that $(L_t^{(1)})$ and $(L_t^{(2)})$ are $\perp$ to I, whence the result will follow.

2.2. The jump in A.

By definition of accessibility

$$[\![\tau_A]\!] \subseteq \bigcup_n [\![\tau^n]\!] \cup E$$

where E is evanescent and the τ^n's are predictable.

Set

$$E_n = \{\tau^n = \tau^1\} \cup \ldots \cup \{\tau^n = \tau^{n-1}\} \quad .$$

By Theorem 5.26 in Elliot (1982) (p.44), $E_n \in F_{\tau-}$. Hence, by Theorem 5.25 in the same book, $\tau^n_{\tilde{E}_n}$ is a predictable stopping time. Hence, by using an increasing sequence of stopping times increasing to $\tau^n_{\tilde{E}_n}$, it is easy to see that

$$\Delta N_{\tau^n_{\tilde{E}_n}} I_{\{\tau^n_{\tilde{E}_n} \leq t\}}$$

is a martingale $\perp G$. Hence, as A is the disjoint union (up to a null-set) of $\{\tau^n_{\tilde{E}_n} = \tau\}$ (note that $P(\tau^n_{\tilde{E}_n} = \tau_B) = 0$ by definition of predictability and total inaccessibility) and as

$$\Delta N_\tau I_{\{\tau = \tau^n_{\tilde{E}_n}\}} = \Delta N_{\tau^n_{\tilde{E}_n}} ,$$

it follows that

$$\begin{aligned} N_t^{(1)} &= I_A \Delta N_\tau I_{\{t \geqq \tau\}} \\ &= \sum_n \Delta N_\tau I_{\{t \geqq \tau\}} I_{\{\tau = \tau^n_{\tilde{E}_n}\}} \\ &= \sum_n \Delta N_{\tau^n_{\tilde{E}_n}} I_{\{t \geqq \tau^n_{\tilde{E}_n}\}} \end{aligned}$$

is a martingale $\perp$ to G.

2.3. The jump in D : conclusion for $(L_t^{(1)})$.

Set

$$N_t^{(2)} = N_t^d - N_t^{(1)}$$
$$= N_t^d - I_A \Delta N_\tau I_{\{t \geqq \tau\}}$$

$(N_t^{(2)})$ is a martingale, $\perp G$ which jumps at τ_B . For a given (G_t) stopping time σ, we have

$$\{\tau_B = \sigma\} \subseteq C \cup E \quad (P(E) = 0) \quad ,$$

hence

$$P(\tau_D = \sigma < \infty) = 0 \quad .$$

By using Theorem 6.46 (p.61) in Elliot (1982), it is easily seen that for all (G_t)-martingales (K_t) :

$$\Delta K_{\tau_D} = 0 \quad .$$

This leads to $(L_t^{(1)})$ being orthogonal to $\mathrm{stab}_M(G)$, where (π_p^* being the dual predictable projection)

$$L_t^{(1)} = A_t - \pi_p^*(A_t) \quad ,$$

with

$$A_t = I_D \Delta N_\tau I_{\{t \geqq \tau\}} \quad .$$

To see this, note that the set J of martingales of the form

$$\left(\sum_{i=1}^{n} I_{E_i} M_{t \wedge \sigma_i}^i \right)$$

(where the (M_t^i)s are (G_t)-martingales, $E_i \in F_0$, the σ_i s are (F_t)-stopping times and n is arbitrary) is dense inn I. This is because it is a linear space which is closed under stopping; the set of limits of H^1-convergent sequences on this space is also closed under stopping and therefore stable.

Now of $(K_t) \in I$, there is a sequence in J which converges to (K_t) in M, and a subsequence $(K_t^{(\ell)})$ of this converges also for all t, P-a.s. $(K_\infty^{(\ell)}$ can be chosen to converge P-a.s., the rest follows by right continuity of the sample paths). Hence, for a given stopping time σ,

$$\Delta K_\sigma^{(\ell)} \to \Delta K_\sigma \qquad \text{P-a.s.}$$

However, for fixed ℓ,

$$\Delta K_\sigma^{(\ell)} = \sum_{i=1}^{n} I_{E_i} \; \Delta M_\sigma^i \; I_{\{\sigma_i \geqq \sigma\}} \qquad .$$

In the case of τ_D, this gives that

$$\Delta K_{\tau_D}^{(\ell)} = 0 \qquad ,$$

whereby

$$\Delta K_{\tau_D} = 0$$

for all elements of $\text{stab}_1(G)$. Hence, by Corollary (2.27) (p.35) in Jacod (1979), $(L_t^{(1)})$ is orthogonal to $\text{stab}_1(G)$.

2.4. The jump in C .

Set

$$N_t^{(3)} = N_t^{(2)} - L_t^{(1)}$$
$$= N_t^d - \dot{\sum} I_{A\cup D} \Delta N_\tau I_{\{t \geqq \tau\}} \qquad .$$

This is again a martingale, $\perp G$, which jumps at τ_C .

Set

$$C = \bigcup_n \left\{ \tau_B = \sigma_n^* < \infty \right\} \qquad ,$$

where the σ_n^* are (G_t)-stopping times. Set

$$E_n = \{\sigma_n^* = \sigma_1^*\} \cup \ldots \cup \{\sigma_n^* = \sigma_{n-1}^*\}$$

and let σ_n^{**} be the restriction of σ_n^* to $\tilde{E}_n$. Obviously, $\tilde{E}_n \in G_{\sigma_n}$, so that σ_n is a (G_t)-stopping time. In addition, C is obviously a disjoint union of the sets $\{\tau_B = \sigma_n^{**} < \infty\}$. Now write (by Theorem 5.16 (p.42) in Elliot (1982))

$$\sigma_n^{**} = \sigma_n \vee \sigma_n^{***} \qquad ,$$

where σ_n and σ_n^{***} are, respectively, the totally inaccessible and the accessible parts of σ_n^{**} <u>with respect to the filbration (G_t)</u> .

Now,

$$[\![\sigma_n^{***}]\!] \subseteq \bigcup_k [\![\eta_k]\!] \qquad ,$$

where the η_k s are predictable stopping times with respect to (G_t) and therefore (F_t) . As

$$[\![\tau_B]\!] \cap [\![\eta_k]\!]$$

is evanescent by definition of total inaccessibility, it follows that

$$C = \bigcup_n \{\tau_B = \sigma_n\} \cup E \; , \quad P(E) = 0 \qquad ,$$

the union being disjoint.

Since σ_n is (G_t)-totally inaccessible, the process

$$K_t^{(n)} = A_t^{(n)} - \pi_p^*(A^{(n)})_t \qquad ,$$

where

$$A_t^{(n)} = I_{\{t \geqq \sigma_n\}}$$

and π_p^* is the dual predictable projection with respect to (G_t), is a (G_t)- and therefore (F_t)-martingale. Hence,

$$\Delta N_\tau I_{\{t \geqq \tau\}} I_{\{\tau_B = \sigma_n < \infty\}} = [N^{(3)}, K^{(n)}]_t$$

by the results (2.21) and (2.29) (p.32-37) in Jacod (1979), and this is a martingale by Corollary (2.27) in the same book and by $(N_t^{(3)}) \perp G$.
Hence

$$N_t^{(4)} = \Delta N_\tau I_{\{t \geqq \tau\}} I_C$$

$$= \sum_n \Delta N_\tau I_{\{t \geqq \tau\}} I_{\{\tau_B = \sigma_n < \infty\}}$$

is a martingale.

2.5. Further study of the jump at $A \cup C$.

Set

$$L_t^{(2)} = N_t^{(1)} + N_t^{(3)} \quad .$$

Since

$$\Delta N_\tau = \Delta L_\tau^{(1)} + \Delta L_\tau^{(2)} \quad ,$$

it follows that $(N_t^d - L_t^{(1)} - L_t^{(2)})$ is continuous, and therefore, by construction, nul. That is,

$$N_t^d = L_t^{(1)} + L_t^{(2)} \quad .$$

Hence, as mentioned in 2.1, it now remains to show that $(L_t^{(2)}) \perp I$.

From 2.2 and 2.4, we have that

$$L_t^{(2)} = \Delta N_\tau I_{A \cup C} I_{\{t \geqq \tau\}} \quad .$$

Consider the following 3 types of sets:

$$\{s < \tau_{A \cup C} \leqq t\} \cap B \ , \quad B \in F_s \quad ,$$

$$\{s < \tau_{A \cup C}\} \cap B \ , \quad B \in F_s$$

and

$$\{t \geqq \tau_{A \cup C}\} \quad .$$

The integral of $L_{\tau_{A \cup C}}^{(2)}$ over each of these sets is 0, and since $(L_t^{(2)})$ is orthogonal to G, the same holds for $I_E L_{\tau_{A \cup C}}^{(2)}$, where E is any element of G_∞. Since $F_{\tau_{A \cup C}-}$ is the smallest σ-field over the algebra of finite disjoint unions of sets on these 3 forms, this yields that

$$E\left(I_E \Delta L^{(2)}_{\tau_{AUC}} \middle| F_{\tau_{AUC}-}\right) = 0$$

for all $E \in G_\infty$. Hence,

$$E\left(\Delta L^{(2)}_{\tau_{AUC}} \middle| G_\infty \vee F_{\tau_{AUC}-}\right) = 0 \quad .$$

However, from what we have seen in 2.3, all martingales $(K_t) \in I$ satisfy that $\Delta K_{\tau_{AUC}}$ is $G_\infty \vee F_{\tau_{AUC}-}$ -measurable. This is because $\{\sigma_i \geqq \sigma\} \in F_{\sigma-}$ for stopping times σ and σ_i, see Elliot (1982), Theorem 5.13 (4), p.41. Hence,

$$E\left(\Delta L^{(2)}_{\tau_{AUC}} \Delta K_{\tau_{AUC}} \middle| F_{\tau_{AUC}-}\right) = 0 \quad ,$$

which yields that $[L^{(2)},K]_t$ is a martingale. $(L^{(2)}_t) \perp I$ follows by Jacod (1979)'s Corollary 2.29 (p.36).

(qed)

Acknowledgements: The author is grateful to Professor Dag Tjøstheim and to two referees on a previous manuscript for valuable advice and comments. Thanks also to Mrs. Gidske Tresselt for typing the manuscript.

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Correction to statistical report no. 15, Dept. of Mathematics, University of Bergen, Norway. December 1988.

Part 1.

p.1: Add at end of first paragraph: "If I is a subset of M, we say that $(N_t)\perp I$ if $(N_t)\perp(L_t)$ for all $(L_t)\in I$ so that (N_tL_t) is locally integrable. Locally integrable here means that there is a sequence of stopping times η_n, $\eta_n\to\infty$ as $n\to\infty$, so that $N_{t\wedge\eta_n}L_{t\wedge\eta_n}$ is integrable for each n and t."

p. 3: "measue" should be "measure". $F_t^{X,Y}$ should be $F_t^{X,Z}$.

Part 2.

p.5: τ is the first jump exceeding 1. The second paragraph (with the decomposition $N_t=N_t^c+N_t^d$) is omitted.

p.6: Last paragraph ("As (N_t^c) is ...") to be replaced by:

"We shall study the jump at time τ in A, C and D separately. From this, (N_t) will naturally be decomposed $N=L_t^{(1)}+L_t^{(2)}$+remainder, $(L_t^{(1)})$ taking care of the jump over D and $(L_t^{(2)})$ of the rest of the jump at time τ. Now let (K_t) be an arbitrary element of I. Local integrablilty of N_tK_t is equivalent to local integrability of $[N,K]_t$, and one is a local martingale if the other so is. We shall show that $[L^{(1)},K]_t=0$ (sect. 2.3), and that if $[N,K]_t$ is locally integrable, then so is $[L^{(2)},K]_t$, and the latter is a local martingale (sect. 2.5). In particular (since (K_t) can be any element of G), it follows that $(L_t^{(1)}+L_t^{(2)})$ is orthogonal to G, whence the same holds for the remainder. Being locally bounded, the remainder is also $\perp$ to every element of I, by Thm. 4.7 (p. 117) in Jacod (1979), and in particular to the (K_t) from the beginning of the paragraph. Hence $[\text{remainder},K]_t$ is a local martingale. Hence $[N,K]_t$ is a local martingale whenever it is locally integrable. The result follows."

p.8: Omit first two sentences of sect. 2.3. "This leads to $(L_t^{(1)})$ being orthogonal to $\text{stab}_M(G)$," should be replaced by "This leads to $[L^{(1)},K]_t=0$ for all $(K_t)\in I$,"

p.9: Last sentence replaced by: "Hence, by the proof of Corollary (2.27) (a) (p.35) in Jacod (1979), $[L^{(1)},K]_t=0$ for all t.

p.10: Omit first two sentences of sect. 2.4. "filbration" should be "filtration".

(continued next page)

p.11: Replace $N_t^{(3)}$ by N_t. Replace $N_t^{(4)}$ by $N_t^{(3)}$.

p.12: Second and third sentence of sect. 2.5 should be omitted.

p.13: Last two sentences of sect. 2.5 should be replaced by "Since $[L^{(2)},K]_t = \Delta L^{(2)}_{\tau_{AUC}} \Delta K_{\tau_{AUC}} I_{\{t \geq \tau_{AUC}\}}$, it follows that if η is some bounded stopping time so that $[L^{(2)},K]_{t\wedge\eta}$ is integrable, then

$$E\left[[L^{(2)},K]_{\tau_{AUC}\wedge\eta} \,\middle|\, F_{\tau_{AUC}-} \cap F_\eta\right] = 0 \quad ,$$

whence $[L^{(2)},K]_{t\wedge\eta}$ is a martingale. Since we have from sect. 2.1 and sect. 2.3 that $[L^{(1)},K]_t = 0$ and $[\text{remainder},K]_t$ is locally integrable, assuming that $[N,K]_t$ is locally integrable implies that $[L^{(2)},K]_t$ is locally integrable, and, by the preceeding, a local martingale. (qed)"

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