MathFinance 346/Stat 391. Winter 2005.

Homework for Lecture 8. Due Wed 3/9.

In addition to this problem, please calculate as many of the analytic expressions for options that are given in Hull as you have time for. (These are not to be handed in).

1. **Simple default**. The problem concerns a defaultable bond. The risk free interest rate is constant at r, so that the money maket bond is given by $B_t = e^{rt}$. We take B_t as numeraire. Meanwhile, the *Venture Company* issues a money market bond with constant interest rate m, so the face value of the company bond is $G_t = e^{mt}$. The bond can be redeemed for its face value so long as the company is solvent. However, the company can go bankrupt, at stopping time τ. If that happens, the company bond becomes valueless. In other words, taking account of the possibility of bankruptcy, the value of the company bond is actually

$$G_t = e^{mt} I\{\tau > t\}$$

Note that the following three problems are mostly independent of each other.

(a) Assume that $m > r$. What is the risk neutral distribution of τ?

(b) Call the discounted bond $\tilde{G}_t = G_t/B_t$, and call the risk neutral measure P^*. Show that $P^*(\tau < \infty) = 1$. Is it the case that

$$E^*(\tilde{G}_\tau) = \tilde{G}_0 \ ?$$

Does your answer shed any light on whether $\tilde{G}$ is a P^* martingale? Why or why not? And is $\tilde{G}$ in fact a martingale?

(c) Suppose that the Venture Company also has a stock S_t, which follows (under the true probability distribution P)

$$dS_t = \mu S_t dt + \sigma S_t dW_t. \tag{1}$$

until time τ, at which point it becomes valueless, Consider a European call $\eta = (S_T - K)^+$, with fixed maturity T. Can η be replicated in securities B, G and S? If so, what is the price for η at time 0, and what is the hedging strategy?