

MathFinance 345/Stat 390. Autumn 2005.

Homework 1. Due Wed 10/6.

1. Shreve, exercise 1.2.

2. Shreve, exercise 1.6.

3. **A Swap contract.** The contract calls for the following:

(a) The buyer X pays the seller Y an amount q to enter into the contract at time 0.

(b) The seller agrees to exchange 1 share of asset A for 1 share of asset B at time 1.

The share prices of assets A and B at times $t = 0$ and $t = 1$ are S_t^A and S_t^B , respectively. As in all such problems, the share price of the underlying assets at the termination time $t = 1$ are subject to uncertainty, but the model is not necessarily binomial. Assume that there is a riskless asset B with rate of return r . Determine the fair market value q of the contract in two ways:

(a) by an arbitrage argument; and

(b) using the Fundamental Theorem.

4. **An incomplete market.** Consider a market with two assets, bond B and stock S , and three scenarios H , T , and E (for edge). Assume that at time $t = 1$, the share price S_1 is uS_0 , dS_0 , and mS_0 in these respective scenarios, where $u > m > d$.

(a) Show that the market is incomplete.

(b) Exhibit a derivative security for which there is no replicating portfolio in the two assets B and S .

(c) Show that the $t = 0$ market price for the derivative security you found in part (b) is not uniquely determined. (That is, show that there are risk neutral measures for the market that give different prices for the derivative security).

(d) Show that the set of possible market prices of the derivative security in (b) is an interval of real numbers.

Please read the entire Chapter 1 in Shreve before the next class.