

MathFinance 345/Stat 390. Autumn 2004.

Homework 2. Due Wed 10/14.

1. **Numeraire and risk neutral measures.** Consider a market with time periods $t = 0$ and $t = 1$. There are two assets, a risk free bond B_t and a bond Λ_t paying random interest. By this we mean that $B_0 = \Lambda_0 = 1$, while $B_1 = e^r$ and $\Lambda_1 = e^R$, where r is constant and known in advance, while R is random. We suppose that R can take two values ρ_1 and ρ_2 , where $0 < \rho_1 \leq \rho_2$.

(a) On the basis of Lecture 1, state the conditions on ρ_1 and ρ_2 for there to be no arbitrage. Also, taking B as numeraire, give the risk neutral measure π .

(b) Take instead Λ as numeraire. Determine whether the conditions for no arbitrage are the same as in (a). If not find the conditions in this case. Also, assuming no arbitrage, find the risk neutral measure Π corresponding to this numeraire. Find the conditions under which $\Pi \neq \pi$.

2. **Put call parity.** A T period market is given with securities B_t (money market bond, $B_t = \exp\{r_1 + \dots + r_t\}$ where r_i is random interest at time i), Λ_t (zero coupon bond, which pays $\Lambda_T = 1$ dollar at time T), S_t (stock), C_t (call option, paying $(S_T - K)^+$ dollars at T), and P_t (put option, paying $(K - S_T)^+$ dollars at T). Construct a replicating portfolio in P_t using the other assets. (We do not here assume a binomial model).

3. **Calculating with conditional expectations.** Consider the calculational example of conditional expectation given in p. 16 this lecture. Consider a partition $\mathcal{Q} = \{A, D\}$, where $D = \{\omega_3, \omega_4, \omega_5\}$. Find $E(X|\mathcal{Q})$ from $E(X|\mathcal{P})$, using the law of iterated expectations. (Do not calculate $E(X|\mathcal{Q})$ from first principles, except, if you like, to check your answer).

Please read until the end of Chapter 2 in Shreve. Also, it is a good exercise to do as many of the problems in Shreve as you can, even if we do not ask you to hand them in. Also, make sure you understand the connection between the description in Shreve's book and the one in the transparencies.