Quadratic Voting

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I. What's Wrong With ...

Exhibit 1: The Northern Spotted Owl

Spotted Owls

The northern spotted owl was listed as a threatened species under the Endangered Species Act throughout its range of northern California, Oregon and Washington by the United States Fish and Wildlife Service on June 23, 1990[7] citing loss of old-growth habitat as the primary threat. The USFWS previously reviewed the status of the northern spotted owl in 1982, 1987 and 1989 but found it did not warrant listing as either threatened or endangered. Logging in national forests containing the northern spotted owl was stopped by court order in 1991.[2]

In 1990, the logging industry estimated up to 30,000 of 168,000 jobs would be lost because of the owl's status, which agreed closely with a Forest Service estimate.[9] Harvests of timber in the Pacific Northwest were reduced by 80%, decreasing the supply of lumber and increasing prices.[2] However, jobs were already declining because of dwindling old-growth forest harvests and automation of the lumber industry.[9]

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What's the Problem?

Two problems, actually:

(1) Highly asymmetric utilities for majority (tree-loving U. S. middle class) and minority (clear-cutting loggers).

(2) Difficulty in assessing utilities (what is the real cost of another lost species?)

Tyranny of the Majority

This was one of the major concerns of the framers of the Constitution in the 1780s. James Madison referred to the idea as "the violence of majority faction" in The Federalist Papers, for example Federalist 10. The phrase "tyranny of the majority" was used (possibly for the first time) by John Adams in 1788. The issue was also discussed at length by Edmund Burke in Reflections on the Revolution in France (1790) .

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Exhibit 2: Retention of Chicago Judges

Cook County Circuit Judge Matthew Coghlan loses retention bid

Cook County Circuit Judge Matthew E. Coghlan. | cookcountyjudges.org

What's Wrong with Democracy?

Exhibit 2: Retention of Chicago Judges

By Carlos Ballesteros @ballesteros_312 | email

It's been 28 years since a Cook County judge has been voted out of office.

Circuit Court Judge Matthew Coghlan broke that streak on Tuesday, notching 52.4 percent affirmative votes with 79 percent of precincts reporting — well shy of the 60 percent "yes" votes needed for Cook County judges to keep their seats.

Coghlan was first elected to the bench in 2000. He couldn't be reached for comment.

Scrutiny of Coghlan's background was spearheaded by reports from Injustice Watch, an online news outlet, that were published in the Chicago Sun-Times. Those reports fueled calls from progressive organizations and lawyer groups to oust the 18-year incumbent.

RELATED

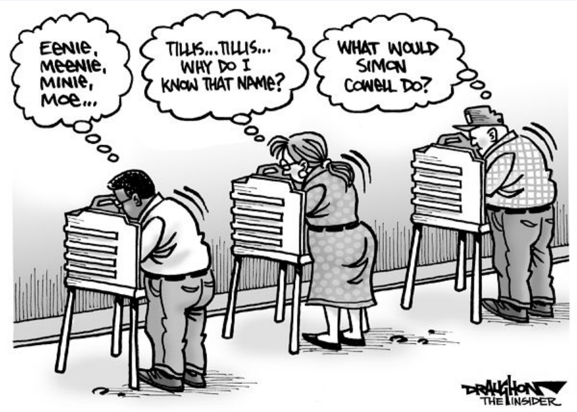
• 2018 Cook County judicial election results

In June, two men exonerated after 23 years in custody sued Coughlan, alleging that — during his time as a county prosecutor — he helped frame them for murder in collusion with disgraced Chicago police Detective Reynaldo Guevara. Coghlan denied the allegations in a court filing in August.

In September, following mounting pressure for progressive groups, the Cook County Democratic Party rescinded its recommendation for Coghlan's reelection.

What's the Problem?

Noise: Voters have either near-zero utilities or no incentive for properly assessing their utilities, so their votes merely introduce noise into the vote totals.



II. Quadratic Voting

Proposal: (Weyl 2012)
Allow voters to buy arbitrary numbers of votes, which they can then cast for either side.
To buy v votes, a voter must pay v^2 dollars.

Quadratic Voting: Is It Legal?

The Twenty-fourth Amendment to the United States Constitution prohibits both Congress and the states from conditioning the right to vote in federal elections on payment of a poll tax or other types of tax. The amendment was proposed by Congress to the states on August 27, 1962, and was ratified by the states on January 23, 1964.

So I think the answer is NO.

Really?

ADAM ROGERS SCIENCE 04.16.19 07:00 AM

COLORADO TRIED A NEW WAY TO VOTE: MAKE PEOPLE PAY—QUADRATICALLY

In a modified version of quadratic voting, Colorado legislators each got 100 virtual tokens to buy votes on a number of measures. "There was a pretty clear signal on which items, which bills, were the most important for the caucus to fund," says one state rep.

JOE AMON/THE DENVER POST/GETTY IMAGES

Quadratic Voting: Rationale

Assume that each voter has a value $u \in \mathbb{R}$. If the election goes in her favor, her utility is $+u$; if it goes against her, her utility is $-u$. The marginal benefit to this voter of an additional unit of vote is

$$2u \times \text{marginal pivotality},$$

where marginal pivotality is the perceived probability that an additional unit of vote will sway the election. Maximizing expected utility - vote cost leads to the rule

$$v = Cu$$

where $C =$ marginal pivotality. Thus, if all voters have the same C, the vote total will be

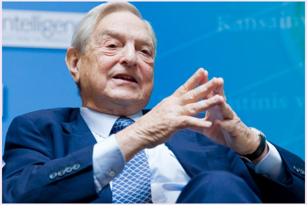
$$C \sum_i u_i.$$

Eeeck!!!

But, Dr. Weyl, don't you think that the wealthiest 1% exercise enough influence on our elections? And now you want to allow them to buy votes?

Mr. Soros Plots His Next Move...

Gotta think that there will be at least 50,000,000 people who will shell out \$1 to vote. Hell, just think of how many idiots click that \$1 box on their tax forms? .... Now , I wonder, if put up \$100,000,000 how many votes will it buy me?



On second thought, maybe I should just hire some Russians?

But there is a problem...

But there is a problem...

III. Bayes-Nash Equilibria

Rationale for QV: A voter with value $u \in \mathbb{R}$ will maximize her utility by purchasing $v = Cu$ votes where C is her marginal pivotality=her perceived probability that an additional unit of vote will turn the election. Thus, if all voters have the same marginal pivotality C, the vote total will be

$$C \sum_i u_i.$$

But why should they? – In fact, they won't: a voter with a very large value u knows that she will buy a larger number of votes than a vote with a small u, and thus will have a different (although perhaps only a slightly different) personal probability distribution for the vote total.

Model Assumptions

Assumption 1: There are N voters, each with a value u_i. These are drawn independently from a continuous probability distribution F with a C^∞, strictly positive density f on a finite closed interval $[\underline{u}, \overline{u}]$.

Assumption 2: Each voter knows his own value u_i but not the values of any of the other voters. However, each voter knows the sampling distribution F.

Assumption 3: Each voter chooses a number $v_i \in \mathbb{R}$ of votes to buy, and pays v_i^2 for these. The payoff to the voter is then

$$\Psi(V)u_i \quad \text{where} \quad V = \sum_{i=1}^{N} v_i$$

and the payoff function Ψ is a smoothed version of $I_{[0,\infty)} - I_{(-\infty,0)}$, that is, $\cdots$

Model Assumptions

Payoff Function:

$$\Psi(x) = -1 + 2 \int_{-\infty}^{x} \psi(y)\, dy$$

where ψ is an even, C^∞ probability density with support $[-\delta, \delta]$ such that ψ is positive and strictly increasing on $(-\delta, 0]$.

Rationale for smoothing: If the vote total is near zero, the payoff to the winning side is reduced because of (i) the possibility of a recount, or (ii) the possibility of having to form coalitions.

Bayes-Nash Equilibria

Definition: A type-symmetric, pure-strategy Bayes-Nash equilibrium is a function $v(u)$ such that for every $u \in [\underline{u}, \overline{u}]$, the value $v(u) = v$ maximizes

$$E[u\Psi(S_n + v)] - v^2$$

where $S_n = \sum_{i=1}^n v(U_i)$ is the one-out vote total and $n = N - 1$. (In particular, $U_1, U_2, \cdots, U_n$ are i.i.d. with density f.)

Theorem: For each $N > 1$ there is at least one increasing, type-symmetric, pure-strategy Bayes-Nash equilibrium.

Note: No one would ever pay more than $2\max(|\underline{u}|, |\overline{u}|)$ for votes, since this is the maximum possible resulting change in utility. Therefore, every Bayes-Nash equilibrium $v(u)$ must satisfy

$$\|v\|_\infty \leq \sqrt{2\max(|\underline{u}|, |\overline{u}|)}.$$

Bayes-Nash Equilibria: Euler-Lagrange Equation

Necessary Condition for a Nash Equilibrium:

$$v(u) = \underbrace{\frac{E\left[\psi\left(S_n + v(u)\right)\right]}{2}}_{\text{marginal pivotality}} u.$$

The proof is standard and easy.

Difficulty: The Euler-Lagrange equation can't be "solved" in any explicit fashion because the expectation involves the unkown function $v(u)$:

$$S_n = \sum_{i=1}^{n} v(U_i).$$

Bayes-Nash Equilibrium: Balanced Populations

Theorem A: Assume that the sampling distribution F has mean $\mu = 0$ and variance σ^2. Then there exist constants $\varepsilon_N \to 0$ such that for all sufficiently large N,

(A) every Bayes-Nash equilibrium $v(u)$ is C^∞ on $[\underline{u}, \overline{u}]$;

(B) every Bayes-Nash equilibrium $v(u)$ satisfies the approximate proportionality rule

$$\left| \frac{v(u)}{p_N u} - 1 \right| \leq \varepsilon_N \quad \text{where} \quad p_N = \frac{1}{2^{\frac{3}{4}} \sqrt{\sigma} \sqrt[4]{\pi N}};$$

(C) every Bayes-Nash equilibrium $v(u)$ satisfies

$$P\left\{ \left(\sum_{i=1}^{N} U_i \right) \left(\sum_{i=1}^{N} v(U_i) \right) < 0 \right\} < \varepsilon_N$$

Thus, quadratic voting is asymptotically efficient: the "right side" wins with high probability.

Why $1/\sqrt[4]{N}$?

Suppose that everyone had the same marginal pivotality β, so that $v(u) = \beta u$. Then by Euler-Lagrange, coupled with the Local Central Limit Theorem,

$$2\beta = E\psi(V) = E\psi\left(\sum_{i=1}^{N} \beta U_i\right) \approx P\left\{|\sum_{i=1}^{N} U_i| \leq \frac{\delta}{\beta}\right\} \approx \frac{2\delta}{\beta\sigma\sqrt{2\pi N}}.$$

Bayes-Nash Equilibria: Unbalanced Populations

The characterization of Bayes-Nash equilibria when F has mean $\mu > 0$ depends on an auxiliary optimization problem.

Optimization Problem: Determine $\xi > \delta$ and $w \in [-\delta, \delta]$ such that

$$(1 - \Psi(w))\,|\underline{u}| = (\xi - w)^2 \quad \text{and}$$
$$(1 - \Psi(w'))\,|\underline{u}| < (\xi - w')^2 \quad \text{for all } w' \in [-\delta, \delta] \setminus \{w\}$$

Proposition: If $\sqrt{2}\delta < \max(|\underline{u}|, |\overline{u}|)$ then there is a unique solution to the Optimization Problem.

Bayes-Nash Equilibria: Unbalanced Populations

Theorem B: Assume that the sampling distribution F has mean $\mu > 0$, and assume that the Optimization Problem has a unique solution (ξ, w). Then $\exists \zeta > 0$ such that $\forall \varepsilon > 0$ and any Bayes-Nash equilibrium $v(u)$,

(a) $v(u)$ has a jump discontinuity at $u = u_*$, where

(b) $|u_* - (\underline{u} + \zeta N^{-2})| < \epsilon N^{-2}$;

(c) $v(u)$ is infinitely differentiable for $u \in (u_*, \overline{u}]$;

(d) $v(u) = (\xi/(N\mu))u + o(1/N)$ for $u > u_*$; and

(e) $|v(u) + \xi - w| < \epsilon$ for $u \in [\underline{u}, u_*)$.

Amplification: (d) and the WLLN implies that with probability $1 - O(1/N)$ the vote total will be within ε of ξ. (e) implies that with probability $O(1/N)$ there will be an extremist with value $u \in [\underline{u}, u_*]$ who will buy enough votes to move the vote total to $\approx w$.

Unbalanced Populations ($\mu > 0$) : Discussion

1. Why don't typical voters buy more votes?

If with high probability the vote total $V > \xi' > \xi$ then not even an individual with the lowest possible value $\underline{u}$ would find it worthwhile to buy enough votes to move the vote total from ξ' to some $w \in [-\delta, \delta]$, because the Optimization Problem implies that the cost of buying enough votes to do so would exceed the benefit in utility:

$$(1 - \Psi(w))|\underline{u}| < (\xi' - w)^2 \ \ \forall\, w \in [-\delta, \delta].$$

So What?

But then Hoeffding's Inequality would imply that $P\{V \in [-\delta, \delta]\}$ is exponentially small, and so the marginal pivotality would be exponentially small, and so most voters would only buy $e^{-\beta N}$ votes. Thus, the vote total would be near 0!

Unbalanced Populations ($\mu > 0$) : Discussion

2. Why don't typical voters buy fewer votes?

Voters in the "bulk" of the distribution F will all have similar marginal pivotalities, so the vote function must satisfy $v(u) \approx \beta u$ except in the extreme tails of F. But since $\mu > 0$ it then follows that with high probability $V \approx \beta\mu N > 0$. Now

(1) Suppose that $0 < \beta\mu N \leq \delta$. Then with high probability $0 \leq V \leq \delta$, and so the marginal pivotality $\beta = E\psi(V)$ would remain bounded away from 0, forcing $\beta\mu N \to \infty$. So this is impossible.

(2) Suppose that $\delta < \beta\mu N < \xi$. Then by the Optimization Problem a nonvanishing fraction of the population – those with values in the lower range of the sampling distribution – would find it advantageous to unilaterally buy enough votes to move the vote total from $\beta\mu N$ to some $w \in [-\delta, \delta]$, and therefore would do so. This would force $V \leq \delta$ with high probability, a contradiction since $V \approx \beta\mu N$.

Key Idea 1: Approximate Consensus

Consensus Lemma: For any $\varepsilon > 0$ and any $\delta > 0$, if N is sufficiently large then in any Bayes-Nash equilibrium $v(u)$,

$$1 - \varepsilon < \frac{E\psi(v(u) + S_n)}{E\psi(v(u') + S_n)} < (1 - \varepsilon)^{-1}$$

for any two values u, u' that are not within distance δ of either $\underline{u}$ or $\overline{u}$. (Here S_n is the vote total for a sample of size $n = N - 1$.)

Crude Argument: Knowing that my value is u gives me almost no information about the empirical distribution of the values U_i, including its tail, unless u is so close to one of the extremes $\underline{u}, \overline{u}$ that I can deduce that with high probability there is no one else in the sample above (or below) me.

Consensus

Multinomial Sampling: Consider random sampling from set $\{A, B, C\}$ with probabilities $\{p_A, p_B, p_C\}$. Let N_A, N_B, N_C be the numbers of A, B, C's in sample of size $n = N_A + N_B + N_C$.

My Type: A.
Your Type: B.

Let's compare our respective conditional distributions on (N_A, N_B, N_C).

$$P^{Me}\{N_i = n_i \, \forall \, i = A, B, C\} = \frac{(n-1)!}{(n_A - 1)! n_B! n_C!} p_A^{n_A - 1} p_B^{n_B} p_C^{n_C}$$

$$P^{You}\{N_i = n_i \, \forall \, i = A, B, C\} = \frac{(n-1)!}{n_A! (n_B - 1)! n_C!} p_A^{n_A} p_B^{n_B - 1} p_C^{n_C}$$

Consensus

Multinomial Sampling: Consider random sampling from set $\{A, B, C\}$ with probabilities $\{p_A, p_B, p_C\}$. Let N_A, N_B, N_C be the numbers of A, B, C's in sample of size $n = N_A + N_B + N_C$.

My Type: A.
Your Type: B.

Let's compare our respective conditional distributions on (N_A, N_B, N_C).

So$\cdots$

$$\frac{P^{Me}\{N_i = n_i \,\forall\, i = A, B, C\}}{P^{You}\{N_i = n_i \,\forall\, i = A, B, C\}} = \frac{n_A p_B}{n_B p_A}.$$

The probability that in a random sample of size n this ratio differs from 1 by more than ε is exponentially small.

Consensus

Multinomial Sampling: Now consider random sampling from set $\{A, B, C\}$ with probabilities $\{1/N^2, p_B, p_C\}$. Let N_A, N_B, N_C be the numbers of A, B, C's in sample of size $n = N_A + N_B + N_C$.

My Type: A.
Your Type: B.

Let's compare our respective conditional distributions on (N_A, N_B, N_C).

$$P^{Me}\{N_A = 1\} \approx 1$$
$$P^{You}\{N_A = 0\} \approx 0.$$

Our conditional distributions are nearly singular!

Key Idea 2: Anti-Concentration

Proposition: $\forall \varepsilon > 0$ and $\forall C < \infty$ there exist $C' > 0$ and $n' < \infty$ such that if $n \geq n'$ and $Y_1, Y_2, \cdots, Y_n$ are i.i.d. random variables such that

$$E|Y_1 - EY_1|^3 \leq C \mathrm{var}(Y_1)^{3/2} \quad \text{and} \quad \mathrm{var}(Y_1) \geq C'/n$$

then for every interval J of length at least 1,

$$P\left\{\sum_{i=1}^{n} Y_i \in J\right\} \leq \varepsilon |J|.$$

(Berry-Esseen Theorem)

Consequence: This limits the number of votes that a voter will buy in equilibrium, because it implies that large numbers of votes will reduce the marginal pivotality of each voter.

Discontinuities

Proposition: There exists $\Delta > 0$ such that for all large N, at any point u_* of discontinuity of a Bayes-Nash equilibrium,

$$\limsup_{u \to u_*} |v(u) \geq \Delta.$$

Consequence: Discontinuities can occur only in the extreme tails of the distribution F.

Discontinuities

Proposition: There exists $\Delta > 0$ such that for all large N, at any point u_* of discontinuity of a Bayes-Nash equilibrium,

$$\limsup_{u \to u_*} |v(u) \geq \Delta.$$

Proof: Assume $u_* > 0$, and let v_+, v_- be the right- and left- limits of $v(u)$ as $u \to u_*$. Must have

$$2v_+ = E\psi(v_+ + S_n)u \quad \text{and}$$
$$2v_+ = E\psi(v_+ + S_n)u.$$

Subtract:

$$2v_+ - 2v_- = u \int_{v_-}^{v_+} E\psi'(t + S_n)\, dt.$$

Mean Value Theorem implies that $\exists \tilde{v} \in [v_-, v_+]$ such that

$$E\psi'(\tilde{v} + S_n) = 2.$$

Discontinuities

Proposition: There exists $\Delta > 0$ such that for all large N, at any point u_* of discontinuity of a Bayes-Nash equilibrium,

$$\limsup_{u \to u_*} |v(u) \geq \Delta.$$

But this implies that the distribution of $\tilde{v} + S_n$ is highly concentrated in $[-\delta, \delta]$; consequently, for suitable $C > 0$,

$$E\psi(\tilde{v} + S_n)u \geq C,$$

and this implies that the marginal pivotality at u_* is bounded away from 0.

More Interesting Proposals from Dr. Weyl

E. Glen Weyl & Eric Posner

Authors, *Radical Markets: Uprooting Capitalism and Democracy for a Just Society* ● Finance

● At least 16 startups or blockchain projects began this year based in part on ideas in their book.

Weyl, an economist who's a principal researcher at Microsoft Research New York City, and Posner, a professor at the University of Chicago Law School, argue in *Radical Markets* that we need to harness market forces to achieve social good. They want to fundamentally alter people's relationship with private property by having them put a price tag on everything they own and list it online. If you want to keep your home, for instance, you'd have to make it expensive, which means you'd be willing to pay more property taxes on it. The government could use this revenue to cut other taxes, which would incentivize commercial property owners not to sit on vacant lots. Another idea, quadratic voting, lets people express the intensity of their preferences. Each voter gets the same allotment of credits to buy votes. One vote costs one token, but two votes cost four, five votes cost 25, etc. People who feel strongly about an issue would have to concentrate their tokens. Blockchain startup Eximchain Inc. plans to use a version of

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May 10th 2018

Radical Markets: Uprooting Capitalism and Democracy for a Just Society. By Eric Posner and E. Glen Weyl. *Princeton University Press; 368*