STATISTICS 310: OPTIMIZATION & SIMULATION
PROBLEM SET 3

Problem 1. (A) Write a MATLAB program that implements the simple Gibbs sampler (heat bath algorithm) for the vanilla 2D Ising model (constant coupling $J_{x,y} = 1$ for all neighboring sites x, y and zero external field). This is the probability measure μ_β on configuration space $\{-1,+1\}^V$, where $V = [1,N]^2$, given by

$$\mu_\beta(\sigma) = \frac{e^{-\beta H(\sigma)}}{Z(\beta)}$$

where the Hamiltonian function H is

$$H(\sigma) = -\sum_{x\sim y} \sigma_x \sigma_y.$$

Here the sum is over all edges of the graph $[1,N]^2$. (Thus, each pair x, y of vertices that share an edge is counted just once in the sum.) Assume toroidal boundary conditions, so that vertex $(1,x)$ shares an edge with (N,x), and similarly, $(x,1)$ shares an edge with (x,N).

Run your program for the case $N = 32$ starting from a totally random initial configuration for 10,000 steps at $\beta = .65$ and again at $\beta = .50$ (the critical value is known to be around $\beta = .45$). Plot the configuration (use colors!) after (say) 100, 500, 1000, and 10000 steps. You should see the formation of large blobs of constant spin, but it should be slower and less pronounced at $\beta = .5$.

(B) (Optional) Write a MATLAB program that implements the Swendsen-Wang algorithm. (For this you will need variations of the connected-cluster algorithm that you used on HW 1 for site percolation.) Compare its performance with the Gibbs sampler.

Problem 2. Posted on the course web page is a file *CodedText.txt* containing an English-language text that has been encoded using a simple substitution cypher. Use MCMC (or simulated annealing) to crack the cypher. For this, you will have to invent a suitable *likelihood function*, where the parameter space is the set of all permutations of the symbol set. (HINT: Visit Project Gutenberg.) Your solution should report back what you think is the most likely permutation, and you should email this to Rina in tabular form, along with the first couple of pages of decoded text.