STATISTICS 251
MIDTERM PRACTICE EXERCISES

These problems are mostly adapted from problems that have been given on Stat 251 midterms over the past 5 years. They should be of similar level of difficulty as the problems on Thursday's exam.

Problem 1. 1. Professor Ding tosses a fair coin 100 times. Each time the coin comes up Heads he rolls a fair die. Let X be the number of rolls on which the die comes up 6, and let Y be the number of coin tosses that come up Heads.

(a) Give a formula for the joint distribution of X and Y.
(b) What is $P(X = 10)$? Give the answer in a simple closed form, but do not evaluate factorials.

Problem 2. If a fair die is rolled n times in succession, what is the probability that

(a) no roll shows the same number as the preceding roll?
(b) no roll results in one of the numbers showing on one of the two preceding rolls?

Problem 3. A deck of 52 cards is well-shuffled, and then 26 two-card hands are dealt. Let X be the number of hands in which both cards are of the same suit (suit=$\spadesuit, \clubsuit, \heartsuit, \diamondsuit$).

(a) What is EX?
(b) What is Var(X)?

Problem 4. Workers arrive at the Subaru-Isuzu factory in Lafayette, Indiana each morning in vehicles of different sizes and makes carrying different numbers of workers. Suppose that 1/4 of these vehicles bring just a single worker, 1/4 have two workers, 1/4 have three workers, and 1/4 have four workers. The company statistician, who does not know these fractions, is charged with the task of estimating the average number of workers per car. He arranges to take a random sample of 100 workers at the plant, and asks each of these 100 workers how many fellow workers arrived in the same car. (Hence, all will answer either 0,1,2, or 3.) He then averages these 100 numbers and adds 1. Will this give him a good estimate of the quantity he wishes to estimate? Explain.

Problem 5. A drawer contains two coins. Both are trick coins: one (coin A) is biased so as to come up Heads 70% of the time, while the other (coin B) is biased so as to come up Heads 20% of the time. Suppose that one of the two coins is chosen at random and then tossed repeatedly until the first time a toss results in a Head. If the number of tosses required is 4, what is the conditional probability that if the coin were tossed again it would come up Heads?

Problem 6. Ten diplomats, 5 Russian, the other 5 Ukrainian, arrange themselves randomly around a circular table with 10 seats. What is the probability that no Ukrainian is seated next to another Ukrainian?