STATISTICS 251 SECTION 2 (LALLEY)
HOMEWORK ASSIGNMENT 1
DUE FRIDAY SEPTEMBER 29

Problem 1. An urn contains 8 red balls, 10 white balls, and 12 blue balls, all identical except for color. If 3 balls are selected at random from the urn *without replacement*, what is the probability that

(a) all 3 balls are of the same color?
(b) no two balls in the sample have the same color?

Problem 2. Cards are dealt one at a time (without replacement) from a well-shuffled deck of 52 cards until the first heart ♡ appears.

(A) What is the probability that the first heart appears on the 4th card?
(B) What is the probability that 5 or fewer cards are drawn?
(C) What is the probability that no spades ♠ are drawn?

Problem 3. Urn 1 contains 10 red balls and 10 black balls. Urn 2 contains 10 red balls and 20 black balls. A fair coin is tossed. If it comes up Heads, then 2 balls are drawn at random from Urn 1, without replacement. If the coin comes up Tails then 2 balls are drawn at random from Urn 2, without replacement.

(A) Find an appropriate probability space to describe this experiment.
(B) Calculate the probability that of the two balls selected, one will be red, and the other black.

Problem 4. A ballroom dancing class has 20 men and 24 women. Five pairs are to be chosen, each consisting of one man and one women[1] (no individual can be included in more than one pair, as this would make the dancing too complicated). How many different ways can the 5 pairs be chosen?

Challenge Problem. *(Optional)* How many perfect riffle shuffles of a deck with $2N$ cards are needed to return it to its original configuration? HINT: Label the cards $0, 1, 2, \ldots, 2N-1$ from top to bottom. With each perfect shuffle, the top card 0 and the bottom card $2N-1$ remain fixed. Show that the other cards $k = 1, 2, \ldots, 2N-2$ move according to the rule

$$k \mapsto 2k \mod 2N-1.$$

[1]This restriction is dictated by the rules of competitive ballroom dancing, and is not meant to imply that other sorts of pairing might not lead to interesting dancing.