Suggested reading in the 4th edition of Strang's *Linear Algebra and Its Applications*: Section 2.3.

Suggested problems: From the 8 listed below, please select 6 that you find instructive. Unstarred problems are worth 8 points each; starred problems are worth 12 points each. If you submit solutions to more than 6 problems, we will select the best. The total number of points for this assignment is 64. For you to abide by the University Policy on Academic Honesty and Plagiarism, I expect you to avoid accidental plagiarism by following the collaboration guidelines detailed in the course syllabus. In particular, *you may not refer to notes from discussions with peers or professors while preparing the solutions you plan to submit.*

1. *Strang* Section 2.3: Exercises 8 and 10.

2. *Strang* Section 2.3: Exercises 20 and 24.

3. *Strang* Section 2.3: Exercises 28 and 32.

4. *Strang* Section 2.3: Exercises 34 and 42.

5*. Symmetric matrices and skew-symmetric matrices form two subspaces of the vector space of $n \times n$ matrices. (Recall that an $n \times n$ matrix A is symmetric if $A^T = A$ and is skew-symmetric if $A^T = -A$.) Show that every $n \times n$ matrix can be expressed in exactly one way as the sum of a symmetric matrix and a skew-symmetric matrix. What does this imply about the relationship between the dimensions of these two subspaces and the dimension of the space of $n \times n$ matrices? Check your answer by finding bases for these two subspaces, and counting the number of vectors in each of these bases.

6*. Define the *trace* of an $n \times n$ matrix to be the sum of the entries along the diagonal. So, if a_{ij} is the entry in row i and column j, then $\operatorname{tr} A = \sum_{i=1}^{n} a_{ii}$.

(a) Compute the trace for each of the symmetries of the equilateral triangle: e, r, r^2, f, rf, r^2f. What do you observe?

(b) Prove that $\operatorname{tr}(AB) = \operatorname{tr}(BA)$ for all $n \times n$ matrices A and B. Show it is not true that $\operatorname{tr}(ABC) = \operatorname{tr}(BAC)$ for all $n \times n$ matrices A, B and C.

(c) Prove that $\operatorname{tr}(P^{-1}AP) = \operatorname{tr} A$, for all $n \times n$ matrices A and invertible $n \times n$ matrices P.

7*. Let r be counterclockwise rotation by 120 degrees about the origin in 2-space and let f be reflection across the origin in 2-space.

(a) Consider the equilateral triangle whose centroid is at the origin in 2-space and two of whose vertices are at the points $(1/2, -\sqrt{3}/2)$ and $(1/2, \sqrt{3}/2)$. Find matrices for the six symmetries of the equilateral triangle relative to the (standard ordered) basis $\{(1,0)^T, (0,1)^T\}$ of 2-space. Compute the traces of these matrices.

(b) Let V be an n-dimensional (real) vector space, let B be a basis for V, and let F and G be linear transformations from V to V. Prove that if A is the matrix of F relative to a basis B of $\mathbf{R}^n$ and C is the matrix of G relative to the same basis B, then CA is the matrix of $H = G \circ F$ relative to B.

(c) Consider the equilateral triangle whose centroid is at the origin in 2-space and two of whose vertices are at the points $(1/2, -\sqrt{3}/2)$ and $(1/2, \sqrt{3}/2)$. The vectors from the origin to these two points are linearly independent, hence are a basis for 2-space. Find matrices for the six symmetries of the equilateral triangle relative to this basis of 2-space. Compute the traces of these matrices.