Suggested reading in the 4th edition of Strang's *Linear Algebra and Its Applications*:

Sections 1.5, 1.6, 2.1 and 2.2.

Suggested problems: From the 8 listed below, please select 6 that you find instructive. Unstarred problems are worth 8 points each; starred problems are worth 12 points each. If you submit solutions to more than 6 problems, we will select the best. The total number of points for this assignment is 64. For you to abide by the University Policy on Academic Honesty and Plagiarism, I expect you to avoid accidental plagiarism by following the collaboration guidelines detailed in the course syllabus. In particular, *you may not refer to notes from discussions with peers or professors while preparing the solutions you plan to submit.*

1. *Strang* Section 1.5: Exercise 24 and Section 1.6: Exercise 6.

2. *Strang* Section 1.6: Exercises 48 and 60.

3. *Strang* Section 2.1: Exercises 20 and 28.

4. *Strang* Section 2.2: Exercises 12 and 26 .

5*. Consider the two planes in $\mathbf{R}^3$ whose equations are given below.

$$\begin{aligned} 3x_1 - x_2 - x_3 &= 7 \\ x_1 + 3x_2 - 2x_3 &= -6 \end{aligned}$$

(a) Consider the line of intersection of these two planes. Find a point on the line of intersection, and a vector parallel to the line of intersection.

(b) Use your answer to (a) and calculus to find the point on the line of intersection that is closest to the origin in $\mathbf{R}^3$.

(c) Use your answer to (a) to find an equation of the plane through the origin in $\mathbf{R}^3$ that is perpendicular to the line of intersection.

(d) Use your answer to (c) to find the point on the line of intersection that is closest to the origin in $\mathbf{R}^3$.

6*. (a) Show that if $c_1, c_2, c_3, \dots, c_n$ are distinct real numbers, then the columns of the matrix below are linearly independent.

$$\begin{pmatrix} 1 & c_1 & {c_1}^2 & \cdots & {c_1}^{n-1} \\ 1 & c_2 & {c_2}^2 & \cdots & {c_2}^{n-1} \\ 1 & c_3 & {c_3}^2 & \cdots & {c_3}^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & c_n & {c_n}^2 & \cdots & {c_n}^{n-1} \end{pmatrix}$$

(b) Use (a) to prove that if $(x_1, y_1), \dots, (x_n, y_n)$ are n points in the plane with distinct first coordinates then there is a unique polynomial of degree at most $n-1$ whose graph passes through all n points.

7*. For each of the following statements indicate whether the assertion made is TRUE or FALSE. If you believe the statement to be TRUE, give a short argument to support your claim. If you believe the statement to be FALSE, support your claim with a counterexample or short argument and provide a similar SALVAGE statement which is TRUE.

(a) $(B+C)A = BA + CA$, for any $m \times n$ matrices B and C and any $n \times k$ matrix A.

(b) $A(BC) = (AB)C$, for any matrices A, B and C. (Before answering this question, verify that the product on the left hand side is defined exactly when the product on the right hand side is defined.)

(c) If $AB = 0$ (where A and B are matrices for which the product AB is defined), then $A = 0$ or $B = 0$.

(d) $(AB)^T = A^T B^T$, for any $n \times n$ matrices A and B.

(e) $x^T B$ is a linear combination of the rows of B, for any $n \times m$ matrix B and any (column) vector $x \in \mathbf{R}^n$.

(f) Suppose M has m rows and $m+n$ columns. Partition M into its first m columns N and its last n columns B. If you similarly partition a vector $z \in \mathbf{R}^{m+n}$ into its first m components $x \in \mathbf{R}^m$ and its last n components $y \in \mathbf{R}^n$, then $Mz = Nx + By$.

8*. Let $\zeta = \sqrt[3]{2}$ be the cube root of 2, and let $\mathbb{Q}[\sqrt[3]{2}]$ be the set of all numbers of the form

$$a + b\zeta + c\zeta^2$$

where a, b, c are rational numbers.

(a) Show that $\mathbb{Q}[\sqrt[3]{2}]$ is closed under multiplication and addition.

(b) Find 3×3 matrices U, V, W with integer entries so that multiplication and addition in $\mathbb{Q}[\sqrt[3]{2}]$ are equivalent to multiplication and addition in the set of matrices of the form

$$aU + bV + cW \quad \text{where } a, b, c \text{ are rational.}$$

The matrices U, V, W should correspond to the numbers $1, \zeta, \zeta^2$, respectively. Hint: You should really only have to guess what V should be.