Review for final

Stat 230 Fall 2020 Prof. Jiang

Transformations

Suppose X is a random variable with cdf $F_X(x)$ and $Y = g(X)$.

Find the cdf of Y in terms of F_X.

1 Monotonic transformation: Exponential goodness

Suppose $X \sim \text{Exp}(\lambda)$.

a Find the cdf of $Y = \sqrt{X}$.

$F_Y(y) = 1 - e^{-\lambda y^2}$ $y \geq 0$

b Find the pdf of $Y = \sqrt{X}$.

$f_Y(y) = 2\lambda y e^{-\lambda y^2}$ $y \geq 0$

2 Non-monotonic: Quadratic effect of a predictor

Suppose $X \sim \text{Unif}(-1, 1)$ and $Y = X^2$.

a Find the cdf of Y.

$F_Y(y) = \begin{cases} 0 & y < 0 \\ \sqrt{y} & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases}$

b Find the pdf of Y.

$f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

3 Mixed transformation? Responses are proportions

Suppose $X \sim \text{Unif}(0, 1)$ and $Y = \frac{X}{1-X}$.

a Find the cdf of Y.

$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{y}{1+y} & y \geq 0 \end{cases}$

b Find the pdf of Y.

$f_Y(y) = \begin{cases} \frac{1}{(1+y)^2} & y > 0 \\ 0 & \text{otherwise} \end{cases}$

4 Mixed transformation? Responses are counts of a rare event

Suppose $X \sim \text{Exp}(\lambda)$ and $Y = \lfloor X \rfloor$.

a Find the cdf of Y.

$F_Y(y) = \begin{cases} 0 & y < 0 \\ 1 - e^{-\lambda(\lfloor y \rfloor + 1)} & y \geq 0 \end{cases}$

b Find the pmf of Y.

$f_Y(y) = \begin{cases} e^{-\lambda \lfloor y \rfloor}(1 - e^{-\lambda}) & y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

5 Mixed transformation? Is "lucky or unfortunate"

Suppose $X \sim \text{Unif}(-1, 1)$ and $Y = \begin{cases} -X & X < 0 \\ X & X \geq 0 \end{cases}$.

a Find the cdf of Y.

$F_Y(y) = \begin{cases} 0 & y < 0 \\ y & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases}$

b Find the pdf of Y.

$f_Y(y) = \begin{cases} 1 & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

6 Mixed transformation? "Lucky" "unlucky"

Suppose $X \sim \text{Unif}(-1, 1)$ and $Y = \begin{cases} -1 & X < -0.5 \\ X & -0.5 \leq X \leq 0.5 \\ 1 & X > 0.5 \end{cases}$.

a Find the cdf of Y.

$F_Y(y) = \begin{cases} 0 & y < -1 \\ 0.25 & -1 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$

b Find the pdf of Y.

$f_Y(y) = \begin{cases} 0.5 & y = -1 \\ 0.5 & y = 1 \\ 0 & \text{otherwise} \end{cases}$