Cochran-Orcutt w/ 2 predictors

(see sections 8.3 - 8.4)

① Model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$

② Fit model using ordinary LS to find $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$

③ Suppose

shows autocorrelation

t = time/index

④ suppose D-W statistic is close to zero, with small p-value

⑤ perhaps autocorrelation has this form:

$$\varepsilon_t = \rho\,\varepsilon_{t-1} + w \qquad \text{(see equation 8.2)}$$

⑥ From ①, we can write

$$\varepsilon_t = Y_t - \beta_0 - \beta_1 x_{1t} - \beta_2 x_{2t}$$

$$\varepsilon_{t-1} = Y_{t-1} - \beta_0 - \beta_1 x_{t-1} - \beta_2 x_{2t}$$

and if ⑤ is correct, then

$$Y_t - \beta_0 - \beta_1 x_t = \rho(Y_{t-1} - \beta_0 - \beta_1 x_{1(t-1)} - \beta_2 x_{2(t-1)}) + w_t$$

⑦ C-O transformation

$$\underbrace{(Y - \rho Y_{t-1})}_{Y_t^*} = \underbrace{\beta_0(1-\rho)}_{\beta_0^*} + \overset{\beta_1^*}{\beta_1}\overbrace{(x_{1t} - \rho x_{1(t-1)})}^{x_{1t}^*} + \overset{\beta_2^*}{\beta_2}\underbrace{(x_{2t} - \rho x_{2(t-1)})}_{x_{2t}^*} + w_t$$

$$\Rightarrow Y^* = \beta_0^* + \beta_1^* x_{1t}^* + \beta_2^* x_{2t}^* + w_t$$

⑧ Then $\hat{\beta}_0^* = \hat{\beta}_0(1-\hat{\rho})$, $\hat{\beta}_1^* = \hat{\beta}_1$, and $\hat{\beta}_2^* = \hat{\beta}_2$.