THE UNIVERSITY OF CHICAGO

Department of Statistics

DISSERTATION PROPOSAL

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Geometric Methods in Statistics and Optimization

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ABSTRACT

In this talk, we will talk about three ongoing projects. First, we provide a geometric perspective on some statistical estimation problems which seek linear relations among variables. These problems find affine subspaces from the sample data set that, in an appropriate sense, either best represents the data set or best separates it into components. In other words, statistical estimation problems are optimization problems on the affine Grassmannian, a noncompact smooth manifold that parameterizes all affine subspaces of a fixed dimension. The main objective is to show that, like the Euclidean space, the affine Grassmannian can serve as a concrete computational platform for data analytic problems—points on the affine Grassmannian can be concretely represented and readily manipulated; distances, metrics, probability densities, geodesics, exponential map, parallel transport, etc, all have closed form expressions that can be easily calculated; and optimization algorithms have efficient affine Grassmannian analogues that use only standard numerical linear algebra.

Next, we move on to study complexity of Yates's algorithm, which was first proposed to exploit the structure of full factorial designed experiment to obtain least squares estimates for factor effects for all factors and their relevant interactions. In short it is an organized way to do iterative summation which avoids repeated computation. It was later revealed that several algorithms such as Fast Fourier transform and Walsh transform can also be casted as special cases of Yates's algorithm. We further show that Yates's algorithm can be viewed as a multilinear map, or a tensor network state. Moreover, by using tensor unfolding and complexity bound of rectangular matrix multiplication, we obtain a new complexity bound on Yates's algorithm.

Lastly, we discuss a preliminary work on sum-of-squares (SOS) optimization. Polynomial optimization problems often need a form of nonnegativity certificate, which is NP-hard. A natural relaxation is to consider polynomial that can be expressed as sum-of-squares of polynomials, which can be checked by semi-definite programming. Fixing a basis of monomials with degrees up to half of that of the polynomial, we can establish an isomorphism between SOS polynomials and positive semi-definite matrices, with rank equal to the minimum number of squares. Our goal is to exploit the geometry of positive definite matrices and come up with another optimization approach other than the Lasserre hierarchy under certain problem constraints.
