



The University of Chicago
Department of Statistics

PHD THESIS PRESENTATION

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Random Walk Metropolis Chains on the Hypercube

THURSDAY, June 21, 2012, at 10:00 AM
110 Eckhart Hall, 5734 S. University Avenue

ABSTRACT

We consider the Metropolis algorithm for the distribution $\pi(x) = \theta^{S(x)} (1 + \theta)^{-n}$ on the hypercube $\mathcal{X} = \{0, 1\}^n$, where $S(x)$ is the number of ones in $x \in \{0, 1\}^n$ and $\theta \in (0, 1]$ is a constant. The lazy random walk Metropolis algorithm for this model specifies a Markov chain (X_t) on \mathcal{X} that is known to have cutoff at $\frac{1}{1+\theta}n \log n$ with window size n , a result derived with Fourier analysis by Diaconis and Hanlon (1992) and Ross and Xu (1994). In this work we give a new proof of this result that is purely probabilistic. It uses coupling and a projection to a two-dimensional Markov chain $X_t \rightarrow (S(X_t), d(X_0, X_t))$, where $d(X_0, \cdot)$ is the Hamming distance to the starting state X_0 . Next we generalize this result to a broader class of distributions π on the hypercube. The distributions we consider are also unimodal and radially symmetric. Under certain smoothness conditions on π , we show that when started at an endpoint, the random walk Metropolis algorithm has cutoff of order $n \log n$ in this entire class of distributions.

Information about building access for persons with disabilities may be obtained in advance by calling Matt Johnston at 773.702-0541 or by email (mhj@galton.uchicago.edu).