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PhD Dissertation Presentation

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**Displaced Lognormal and Displaced Heston Volatility Skews:
Analysis and Applications to Stochastic Volatility Simulations**

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ABSTRACT

We analyze the displaced (anti-)lognormal (DL) and displaced (anti-)Heston (DH) volatility skew. In particular, for the displaced lognormal, we prove the global monotonicity of the implied volatility, and an at-the-money bound on the steepness of the downward volatility skews, which therefore cannot reproduce some features observed in the equity market. A variant, the displaced anti-lognormal, overcomes this steepness constraint, but its state space is bounded above and unbounded below. We prove the global monotonicity of its implied volatility too. For the displaced Heston dynamics, we show that the at-the-money slope has the same sign as the displacement. What's more, we give an explicit formula for the DL and DH's short-expiry limiting volatility skew, which allows direct calibration of their parameters to volatility skews implied by market data or by other models. We also analyze the large-expiry limiting volatility of the displaced lognormal and give an asymptotic formula of it in the region of large-strike and fixed-strike respectively.

We propose using the DL/DH dynamics as a control variate, to reduce variance in Monte Carlo simulations of the CEV and SABR local/stochastic volatility models. We give simulation results to show that a carefully constructed control variate can significantly reduce the variance in the Monte Carlo simulations. We further propose a combination of the importance sampling and the control variate to reduce the variance. Numerical simulations show that significant variance reduction can be achieved.

Finally we discuss the convergency of the discretisation schemes of the stochastic processes encountered in the Monte Carlo simulations. Under some regularity conditions, we give a partial strong convergency result for the stochastic volatility process. Moreover, we give a strong convergency result for the mean-reverting CEV process.