



**The University of Chicago**  
**Department of Statistics**  
**Master's Seminar**

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**Failure of the Bootstrap for a Parameter  
on the Boundary of the Parameter Space**

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**ABSTRACT**

This paper is concerned with the failure of the bootstrap procedure for estimation and hypothesis testing when the true parameter is on the boundary or an otherwise irregular point of the parameter space. We provide theoretical motivation from Andrews (2000) as well as a simulation study that demonstrates the bootstrap failure. We then attempt to resolve this issue using subsampling and the  $m$  out of  $n$  bootstrap, running into the need for an appropriate choice for the size of the subsample or  $m$ . We illustrate the behavior of the two techniques in a simulation study about two canonical examples of hypothesis testing problems: testing whether the mean vector of a bivariate normal distribution lies (i) on the non-negative  $x$ -axis and (ii) on the union of the  $x$ - and  $y$ -axes.