



The University of Chicago
Department of Statistics

Master's Seminar

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Tail Index Estimation and ARCH Processes

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ABSTRACT

In the field of Extreme Value Theory an important question is the estimation of the tail index α of distributions of the form $1 - F(x) \sim cx^{-\alpha}$ as $x \rightarrow \infty$. For independent data the classic Hill estimator and other more recent alternatives (Sum Plot and Huisman) are well known to give effective results. Yet they are often carried over to studies with dependent data with no justification where they do not give convincing results. In the following we first review the Hill estimator and its recent alternatives , applying them on independent data. Then we show the conditions under which an ARCH(1) process $X_t = \sqrt{\beta + \lambda X_{t-1}^2} Z_t$ is strictly stationary. The tail index of the stationary distribution (when it exists) is the unique strictly positive solution $\kappa(\lambda)$ to the equation: $h(u) = E[(\lambda Z t^2)^u] = 1$. This allows us to compute precisely the tail index. Having the true value of κ we test the performance of various estimation techniques and compare them to a new sub-sampling technique based on ARCH(1) properties.