PHD SEMINAR ANNOUNCEMENT
Department of Statistics

Likelihood Approach for Monte Carlo integration

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ABSTRACT

Monte Carlo integration is formulated by Kong et al as a statistical model using simulated observations as data. The model makes explicit what information is ignored and what is retained about the baseline measure. The baseline measure is estimated as a discrete measure by maximum likelihood, and integrals of interest are estimated as finite sums. In this thesis, we make contributions to Monte Carlo integration by advancing the likelihood approach in both theory and methodology.

For two different situations where independent draws are generated from multiple distributions, we establish the asymptotic optimality of the maximum likelihood estimator. In the first situation, the normalizing constants of the design distributions are analytically intractable and need be estimated. In the second one, the normalizing constants of the design distributions are given, which impose linear constraints on the parameter space. We also prove that the maximum likelihood estimator has asymptotic variance no greater than the crude Monte Carlo estimator under rejection sampling, and no greater than the ergodic average estimator under independence Metropolis-Hastings sampling.

The likelihood approach is also applicable to Markov chain schemes. We propose statistically efficient estimators and related techniques for our method to be computationally effective under Gibbs sampling and under Metropolis-Hastings sampling. Particularly, the resulting estimator of the normalizing constant of the stationary distribution can converge at a rate faster than the square root rate under Gibbs sampling. We also introduce approximate variance estimators, and note that these estimators reliably quantify Monte Carlo variability in our simulation studies.