The University of Chicago
Department of Statistics
Seminar Series

MICHAEL B. WOODROOFE
Department of Statistics
University of Michigan

Bootstrapping the Grenander Estimator

MONDAY, October 25, 2010, at 4:00 PM
133 Eckhart Hall, 5734 S. University Avenue
Refreshments following the seminar in Eckhart 110.

ABSTRACT

Let $X_1, \cdots, X_n \sim^{\text{ind}} F$, where $F(0) = 0$ and F has a non-increasing density f on $[0, \infty)$, and recall that the Grenander Estimator (NPMLE), $\tilde{f}_n$ say, of f is the left hand derivative of the least concave majorant of the empirical distribution function. If $0 < t_0 < \infty$, f is continuously differentiable near t_0, and $f'(t_0) < 0$, then the distribution, H_n say, of normalized estimation error $n^{\frac{1}{3}}[\tilde{f}_n(t_0) - f(t_0)]$ converges to a (non-normal) limit that depends on f through a scale factor, and bootstrap methods suggest themselves for estimating H_n. Thus let $\hat{F}_n$ be an estimate of F, computed from $X_1, \cdots, X_n$; let $X_1^*, \cdots, X_n^* \sim^{\text{ind}} \hat{F}_n$, given $X_1, \cdots, X_n$; and let H_n^* be the conditional distribution of $n^{\frac{1}{3}}[\tilde{f}_n^*(t_0) - \hat{f}_n(t_0)]$, where $\tilde{f}_n^*$ is the Grenander Estimator computed from $X_1^*, \cdots, X_n^*$. and $\hat{f}_n$ is the density of $\hat{F}_n$ if one exists or a surrogate. The case in which $\hat{F}_n(x) = \tilde{F}_n(x) = \int_0^x \tilde{f}_n dy$ and $\hat{f}_n = \tilde{f}_n$ is studied in detail. A main finding is that H_n^* does not converge in probability to anything in this case. The finding is less than a mathematical proof, because one step in the argument relies on simulation, but the simulation seems conclusive. A similar picture emerges if $\hat{F}_n$ is the empirical distribution function and $\hat{f}_n = \tilde{f}_n$. The bootstrap estimator H_n^* is consistent, however, if $\hat{F}_n$ is a suitably smoothed version of $\tilde{F}_n$.