The University of Chicago
Department of Statistics

Seminar Series

MARTIN WAINWRIGHT
Departments of Statistics, Electrical Engineering
and Computer Sciences, University of California, Berkeley

"Precise Thresholds for Sparsity Recovery in the High-Dimensional and Noisy Setting Using ℓ_1 Relaxations"

MONDAY, January 8, 2007 at 4:00 PM
133 Eckhart Hall, 5734 S. University Avenue
Refreshments following the seminar in Eckhart 110.

ABSTRACT

The problem of recovering the sparsity pattern of an unknown signal arises in various domains, including graphical model selection, signal denoising, constructive approximation, compressive sensing, and subset selection in regression. The standard optimization-theoretic formulation of sparsity recovery involves ℓ_0-constraints, and typically leads to computationally intractable problems. This difficulty motivates the development and analysis of approximate methods; in particular, a great deal of work over the past decade has focused on the use of ℓ_1-relaxations for sparsity recovery.

In this work, we analyze the performance of ℓ_1-constrained quadratic programming, known in the statistics literature as the Lasso, for recovering an unknown signal in p dimensions with at most s non-zero entries based on a set of n noisy observations. Of interest is the number of observations n that are required, as a function of the model dimension p and sparsity index s, for exact sparsity recovery. We analyze this question in the high-dimensional setting, in which both the model dimension p and number of observations n tend to infinity. Our main result is to establish, for a broad class of Gaussian measurement ensembles, precise thresholds on the required growth rate such that for all n above threshold, the Lasso recovers the exact sparsity pattern with probability one, and conversely fails to recover with probability one for all n below threshold.

Please send email to Mathias Drton (drton@galton.uchicago.edu) for further information. Information about building access for persons with disabilities may be obtained in advance by calling the department office at (773) 702-8333.