# The University of Chicago
# Department of Statistics

## Seminar Series

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## "Another Look at Estimation for MA(1) Processes with a Unit Root"

**MONDAY, January 22, 2007 at 4:00 PM**
**133 Eckhart Hall, 5734 S. University Avenue**
*Refreshments following the seminar in Eckhart 110.*

## ABSTRACT


The first-order moving average model or MA(1) is given by $X_t = Z_t - \theta Z_{t-1}$, with independent and identically distributed $\{Z_t\}$. This is arguably the simplest time series model that one can write down. The MA(1) with unit root ($\theta = 1$) arises naturally in a variety of time series applications. For example, if an underlying time series consists of a linear trend plus white noise errors, then the differenced series is an MA(1) with unit root. In such cases, testing for a unit root of the differenced series is equivalent to testing the adequacy of the trend plus noise model. The unit root problem also arises naturally in a signal plus noise model in which the signal is modeled as a random walk. The differenced series follows a MA(1) model and has a unit root if and only if the random walk signal is in fact a constant.

The asymptotic theory of various estimators, including Gaussian maximum likelihood and least squares, has been developed for the unit root case and nearly unit root case ($\theta = 1 - \beta/n, \beta > 0$). Unlike standard $1/\sqrt{n}$-asymptotics, these estimation procedures have $1/n$-asymptotics and a so-called pile-up effect, in which $\mathrm{P}(\hat{\theta} = 1)$ converges to a positive value. One explanation for this pile-up phenomenon is the lack of identifiability of $\theta$ in the Gaussian case. That is, the Gaussian likelihood has the same value for the two sets of parameter values $(\theta, \sigma^2)$ and $(1/\theta, \theta^2\sigma^2)$. It follows that $\theta = 1$ is always a critical point of the likelihood function. In contrast, for non-Gaussian noise, $\theta$ is identifiable for all real values. Hence it is no longer clear whether or not the same pile-up phenomenon will persist in the non-Gaussian case. We consider the asymptotic behavior of different types of least absolute deviation (LAD) estimators of $\theta$ in the non-Gaussian case. Simulation results illustrate the limit theory.

(This is joint work with Jay Breidt, Nan-Jung Hsu, and Murray Rosenblatt.)


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Please send email to Mathias Drton (drton@galton.uchicago.edu) for further information. Information about building access for persons with disabilities may be obtained in advance by calling the department office at (773) 702-8333.