

# The University of Chicago

Department of Statistics

## Seminar

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### “How Many Entries of A Typical Orthogonal Matrix Can Be Approximated By Independent Normals”

**Monday, October 4, 2004 at 4:00 PM**  
**133 Eckhart Hall, 5734 S. University Avenue**

#### ABSTRACT

I will present my solution to the well-known open problem by Diaconis stated as follows: what are the largest orders of  $p_n$  and  $q_n$  such that  $Z_n$ , the  $p_n \times q_n$  left upper block of an  $n$  by  $n$  typical orthogonal matrix  $\Gamma_n$ , can be approximated by independent standard normals? This problem is solved by two different approximation methods.

First, we show that the largest order of  $p_n$  and  $q_n$  are  $o(\sqrt{n})$  in the sense of approximation by the variation norm.

Second, suppose  $\Gamma_n = (\gamma_{ij})_{n \times n}$  is generated by  $\mathbf{Y}_n = (y_{ij})_{n \times n}$  through the Gram-Schmidt algorithm where  $\{y_{ij}; 1 \leq i, j \leq n\}$  are i.i.d. standard normals. We show that the largest order of  $m = m_n$  such that  $\epsilon_n(m) := \max_{1 \leq i \leq n, 1 \leq j \leq m} |\sqrt{n}\gamma_{ij} - y_{ij}|$  goes to zero in probability is  $o(n/\log n)$ .

A history from 1914 to 2003 of the problem from Mechanics, Statistics and Imagine Analysis will also be presented.