The University of Chicago

Department of Statistics

Seminar

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"Validity of the Expected Euler Characteristic Heuristic"

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ABSTRACT

We consider the well-studied problem of approximating the distribution of the maximum of a smooth Gaussian process $(X(t), t \in T)$ where T is a nice subset of Euclidean space, or, more generally an abstract manifold, with or without boundary.

This problem arises in neuroimaging where T is a subset of the brain and the maximum of X corresponds to the likelihood ratio test testing for presence of signal in the brain. It also arises naturally in multivariate analysis – the largest root of a Wishart matrix being a canonical example.

Many approaches have been applied to this problem: volume of tubes, expected Euler characteristic heuristic, and "double sum", to name a few.

Recently, it was proved that the volume of tubes and the expected Euler characteristic (EC) approach are in fact equivalent, when they both can be applied. However, the Euler characteristic approach is based on a heuristic argument and the volume of tubes approach relies on an unverifiable assumption on the covariance function of $X(t)$. In the constant mean and variance case this assumption is essentially equivalent to the existence of an isometric (in L^2) embedding of the parameter space into a finite-dimensional sphere. Further, the best error bounds to date for this method depend on the minimal dimension of this embedding.

In this talk, we use the equivalence of these two approaches and a more careful treatment of the error term in the volume of tubes approach to establish sharp error bounds for any smooth constant mean and variance Gaussian process.

This is part of ongoing joint work with Robert Adler, Satoshi Kuriki and Akimichi Takemura.
