The University of Chicago

Department of Statistics

Seminar

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"Random Walks on Infinite Free Products and Infinite Algebraic Systems of Generating Functions"

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ABSTRACT

A security guard patrols an infinite hallway with an infinite sequence of doors, all of which are initially open. He carries with him a suitcase containing infinitely many keylocks, each with a color $\kappa \in \mathbb{N}$, the colors occurring with relative frequencies $\{p_\kappa\}_{\kappa \in \mathbb{N}}$. At each step of his patrol he selects at random from his suitcase a lock and matching key. If at least one door is locked, and if the color κ of the pair he selects from his suitcase matches that of the *last* lock affixed to a door, then he uses the key to unlock that lock, thereby opening the door, and discards the key and both of the locks. Otherwise, he moves to the next unlocked door in the hallway, attaches the lock, and discards the key.

Problem: *What is the probability that, after n steps, no doors are locked?*
The process described above is a random walk on an infinite free product of finite groups (in this case all copies of the two-element group $\mathbb{Z}_2$). For an aperiodic, irreducible random walk on a finite or countable free product of finite groups, the following is true:

Theorem: The probability pn that the random walk returns to its starting state after n steps satisfies the asymptotic law

$$p_n \sim \frac{C}{R^n n^{3/2}}$$

for some constant $C > 0$, where $R > 1$ is the so-called *spectral radius* of the random walk.

I shall discuss a number of related models where similar 3/2-power laws hold, and I shall outline a proof of the theorem above. The key to the proof is that the Greens function of the random walk can be written as a linear combination of countably many first-passage generating functions which, in vector form, satisfy a functional equation of the form

$$F(z) = zQ(F(z))$$
