ABSTRACT

It is well known that the number of modes of a kernel density estimator is monotone nonincreasing in the bandwidth if the kernel is a Gaussian density. There is numerical evidence of nonmonotonicity in the case of some non-Gaussian kernels, but little additional information is available. The present paper provides theoretical and numerical descriptions of the extent to which the number of modes is a non-monotone function of bandwidth in the case of general compactly supported densities. Our results address popular kernels used in practice, for example the Epanechnikov, biweight and triweight kernels, and show that in such cases non-monotonicity is present with strictly positive probability for all sample sizes \( n \geq 3 \). In the Epanechnikov and biweight cases the probability of nonmonotonicity equals 1 for all \( n \geq 2 \). Nevertheless, in spite of the prevalence of lack of monotonicity revealed by these results, it is shown that the notion of a critical bandwidth (the smallest bandwidth above which the number of modes is guaranteed to be monotone) is still well defined. Moreover, just as in the Gaussian case, the critical bandwidth is of the same size as the bandwidth that minimises mean squared error of the density estimator. These theoretical results, and new numerical evidence, show that the main effects of nonmonotonicity occur for relatively small bandwidths, and have negligible impact on many aspects of bump hunting.

This is a joint work with Peter Hall and Michael Minnotte.