

**The University of Chicago**  
**Department of Statistics**

**Seminar**

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**“Second Order Behaviour of M-Estimators  
in Regression with Long Memory Errors”**

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Please see the following page for abstract.

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# Second Order Behaviour of M-Estimators in Regression with Long Memory Errors

by

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## Abstract

A second order discrete time stochastic process is said to have *long memory* if its auto-correlations decay hyperbolically to zero in the lag parameter. The importance of the long memory in econometrics, hydrology, finance and numerous other physical sciences is abundantly demonstrated in Beran (1992, 1994), Baillie (1996), and references therein.

Consider the problem of estimating the common mean  $m$  based on  $n$  observations from a long memory Gaussian process. There are at least two estimators of  $m$ , namely, the sample median  $M_n$  and the sample mean  $\bar{X}_n$ . The surprising fact is that these two estimators are asymptotically equivalent in the first order, i.e.,  $(M_n - \bar{X}_n)/\sqrt{\text{Var}(\bar{X}_n)} \rightarrow 0$ , in probability, as the sample size  $n \rightarrow \infty$ . This is in *complete contrast* to the i.i.d. or weakly dependent situation where it is known that this sequence of r.v.'s converges in distribution to a normal r.v., with mean 0 and some positive variance. This phenomenon occurs more generally in terms of the other estimators of  $m$ , for long memory moving averages, not necessarily Gaussian, and in general regression models.

This talk will begin by discussing the first order equivalence of a class of M-estimators of the regression parameter to the least square estimator in linear regression models with the long memory moving average errors. Then a higher order asymptotic expansion of a class of these M-estimators is given. It is observed that a suitably standardized difference between an M-estimator and the least square estimator has a limiting distribution.

The primary reason for the above surprise is the *uniform reduction principle* of weighted empirical processes, which will be also discussed.

*This talk is based on some joint work with Liudas Giraitis, Kanchan Mukherjee and D. Surgailis.*

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