New, Improved Confidence Intervals for a Binomial Proportion

Lawrence D. Brown
Wharton School, Univ. of Penn.

Summary

The conventional confidence interval for a binomial proportion is

$$\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n} \ .$$

It is widely recognized that this procedure behaves poorly if p is too near 0 or 1. It has been previously observed, but less widely understood, that the coverage of this procedure can also be seriously less than α even when n is moderately large and p is not extreme. We begin by demonstrating the quirky and unsatisfactory coverage probabilities of this conventional procedure.

Several alternative procedures have been proposed. Chief among these is the procedure which results from inverting the standard Central Limit approximation to the equal tails tests. A recent article by Agresti and Coull (*American Statistician*, 1998, v.52, pps.119-126) contains a nice discussion of this procedure, and attributes it to E. B. Wilson (*JASA*, 1927, v.22, pps.209-212). We'll show that the coverage of this procedure is much more satisfactory than that of the conventional one, even for moderately values of n, although there is still a defect when p is very near 0 or 1. We also propose some minor modifications to that procedure to partially adjust for the performance defect when p is near 0 or 1.

Some other satisfactory procedures are also examined including the equal-tail intervals derived from Jeffreys' prior and a simplified approximation to Wilson's procedure suggested by Agresti and Coull.

We also develop Edgeworth expansions for coverage and second-order expansions for interval length which explain why these procedures, and especially Wilson's, are generally much more satisfactory than the conventional one.

(The above is joint work with T. Cai and A. Das Gupta.)